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Change and Rationality in Macroeconomics and Finance Theory: A New Rational Expectations Hypothesis

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Working Paper No. 8

March 8, 2015

Abstract

We call attention to the class of models that serve as the foundation for the rational expectations hypothesis (REH). Models in this class rule out completely any structural change that cannot be fully anticipated with a probabilistic or other quantitative rule. REH models are abstractions of rational decision-making, but only in a hypothetical world in which participants can fully anticipate when and how they might revise their understanding of the process driving outcomes. We propose a new rational expectations hypothesis (NREH) as a way to represent rational decision- making in real-world markets. NREH builds on the insights of Muth (1961) and Lucas

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(1972, 2001) and imposes internal coherence between the economist's under- standing of outcomes and that of the market. However, like Soros's (1987) conceptual framework, NREH models recognize that any quantitative understanding of the process driving outcomes is necessarily provisional, eventually becomes inadequate, and thus requires revision. Consequently, NREH does so in the context of models that are partly open to unanticipated structural change. NREH models accord participants' expectations an autonomous role in internally coherent models. They also incorporate REH's and behavioral economists' insights about the importance of fundamental and psychological considerations, without presuming that market participants are irrational.

JEL Codes: D8, E3

The authors are grateful to the Institute for New Economic Thinking (INET) for supporting this research. The authors thank Tim Cogley, Bruce Elmslie, and Rob Johnson for insightful comments on earlier drafts of this paper. This paper has also benefited from discussions over many years with George Soros and his insistence on the importance of placing "fallibility" at the center of economic and social analysis. The authors are also indebted to Søren Johansen for reading successive revisions of the paper. His incisive comments led us to reformulate NREH and clarify our comparison of it with REH. Earlier versions of this paper were presented at INET's 2014 Plenary Conference in Toronto, seminars at the University of Copenhagen and the University of New Hampshire, and INET's seminar at Columbia University. We thank the participants, particularly Alex Rosenberg and Joseph Stiglitz, for insightful reactions and comments.

1 Introduction

Prior to the rational expectations hypothesis (REH), economists often portrayed forecast revisions with error-correcting rules, such as adaptive expectations. Muth (1961, pp. 315-316) argued that such rules "do not assume enough rationality": market participants would relate their forecasts to their understanding of "the way the economy works." Muth's seminal insight was that an economist can use his own understanding of the economy, as formalized by his model, to represent that of market participants. Muth formulated this idea by imposing within an economist's model coherence between his understanding of the process driving aggregate outcomes and that of the market (an aggregate of participants). In the early 1970s, Robert Lucas pointed out that imposing such internal coherence represents how a rational, profit-seeking individual understands and forecasts outcomes, conditional on the hypothesis that an economist's model is empirically relevant.

Over the last four decades, macroeconomists have implemented Muth's and Lucas's fundamental insights in the context of a particular class of models. These models, which we have called determinate, characterize any change in the economy's structure with a probabilistic or other quantitative rule. By design, determinate models rule out unanticipated structural change. Imposing internal coherence in them, as REH does, represents decision-making by agents who never revise how they understand and forecast outcomes in ways that the economists did not specify *ex ante* in probabilistic terms.

This key feature of REH models sheds new light on their epistemological flaws and empirical shortcomings. Earlier lines of critique, including our own, have focused on REH representations' lack of realism.[1] But, as Sargent (1993) rightly argues, REH is a bold abstraction. Indeed, any attempt to represent forecasting must necessarily ignore many specific features of how market participants actually forecast outcomes.

We argue in this paper that the fundamental problem with REH is not its lack of realism *per se*, but its reliance on determinate models. To be sure, once determinate models are upheld as the relevant economic theory, REH as a representation of

[1] Earlier critics of REH have argued that profit-seeking participants would not actually forecast according to an economist's model (Frydman and Phelps, 1983; Phelps, 1983), owing to learning and coordination problems (Frydman, 1982, 1983; Evans and Honkophja, 2001, 2013; Guesnerie, 2005, 2013), insufficient capacity to calculate, and psychological biases (Shleifer, 2000; Barberis and Thaler, 2003).

conditionally rational forecasting follows on logical grounds. However, the domain in which REH representations are relevant is severely limited: they are abstractions of rational decision-making, but in a hypothetical world in which agents can anticipate today exactly how they will understand the economy's structure at all other points in time.[2] In this world, there is only one conditionally rational way to understand and forecast outcomes, namely that represented by the joint probability distribution implied by an economist's determinate model.

This conclusion leaves open the question of whether REH models could also serve as abstract approximations of decision-making in real-world markets. It is self-evident that, in pursuit of profits, market participants—particularly in asset markets, revise how they understand outcomes in unanticipated ways.[3] These revisions and other sources of unanticipated structural change are important drivers of outcomes. In Frydman and Goldberg (2014a), we show that in real-world markets, REH models represent decision-making by irrational participants who forego profit-opportunities endlessly.

In this paper, we propose a new rational expectations hypothesis (NREH) as a way to build abstractions of conditionally rational forecasting by individuals who cannot fully anticipate today how they will understand the process driving market outcomes at all other periods. NREH does so by formalizing an economist's own understanding of outcomes with models that are open to unanticipated structural change. Like Soros's (1987) conceptual framework, NREH models recognize that any quantitative understanding of the process driving outcomes is necessarily provisional, eventually becomes inadequate, and thus requires revision.[4] However, unless an economist imposes some constraints on such unanticipated structural change, his models have no implications for time-series data. In Frydman and Goldberg (2007, 2011), we proposed imperfect knowledge economics (IKE) as an approach to building such partly open models.

[2] An individual's understanding of economic outcomes depends in part on his knowledge, which includes formal scientific and other forms of knowledge. REH models, therefore, by ruling out all unanticipated structural change, are abstractions of rational forecasting in hypothetical markets in which knowledge does not grow. As Karl Popper formally showed, "[i]f there is such a thing as a growing human knowledge, we cannot anticipate today what we shall only know tomorrow" (Popper, 1957, xii).

[3] Popper's (1957 and 1982) arguments imply that no one can fully know in advance how their understanding of economic, and more broadly social, outcomes will unfold over time.

[4] For a comparison of Soros's conceptual framework and imperfect knowledge economics, see Frydman and Goldberg (2013a).

An IKE model explores the possibility that although revisions of forecasting strategies and other structural changes cannot be fully anticipated, they may nonetheless exhibit regularities that can be formalized with qualitative constraints that can be imposed *ex ante*. For example, we argue that in many macroeconomic contexts, change in the process underpinning outcomes will tend to be moderate for protracted intervals of time. Although these constraints leave the model open to unanticipated structural change, they leave it only partly open during the intervals in which the constraints are relevant. We show in sections 3 and 5 that such constraints imply distinct qualitative regularities in how outcomes and causal variables move over time in the intervals in which they are imposed—for example, that the inflation rate co-moves either positively or negatively with the interest rate.[5]

A key feature of a partly open model is that it does not specify in advance when intervals of constrained change begin or end. Thus, in sharp contrast to their determinate counterparts, the time-series regularities implied by partly open models are not only qualitative, but also contingent: they leave open *ex ante* when intervals in which the data exhibit distinct regularities begin or end.

NREH builds on Muth's and Lucas's insights about the importance of internal coherence for building models that are compatible with rational decision making. NREH imposes internal coherence, but in an IKE model. As with REH, NREH models represent rational forecasting, conditional on the empirical relevance of the economist's model. However, NREH models are partly open and thus are abstractions of rational decision-making in real-world markets, in which participants revise their understanding of outcomes in unanticipated ways. Moreover, because NREH models imply regularities that are qualitative and contingent, they are compatible with myriad ways in which rational, profit-seeking participants in real-world markets forecast outcomes.

REH models account for market outcomes in terms of fundamental considerations, such as movements in interest rates and income. By imposing internal coherence, NREH models also provide fundamental accounts of outcomes. But NREH's reliance on partly open models reveals that the insights of two other major advances

[5] In Frydman and Goldberg (2013b), we formulate an IKE version of a New Keynesian model that relates an economy's inflation rate to the real interest rate. Although the model is open to unanticipated structural change, its qualitative and contingent constraints on such change imply that the inflation rate will co-move positively with the interest rate during some time intervals and negatively during others. See sections 3 and 5 for an example and further discussion.

in macroeconomics over the last four decades are also important for understanding rational decision-making in real-world markets. One advance is Phelps's et al (1970) research program, which based aggregate relationships on micro-foundations that accord an autonomous role to market participants' expectations. The other is behavioral economists' use of empirical observations in representing individuals' decision-making. Both approaches are compatible with diversity of forecasting strategies.

As with REH, these advances have been implemented in the context of determinate models. However, according expectations an autonomous role in such models necessarily renders them internally incoherent. Consequently, the profession's reliance on determinate models has posed a stark choice for macroeconomists: they can either presume that market participants forego obvious profit opportunities, or impose REH and assume away the importance of autonomous expectations and diversity of forecasting strategies.

By opening macroeconomics to unanticipated structural change, NREH accords participants' forecasting an autonomous role in the context of internally coherent models. Moreover, NREH enables economists to incorporate both fundamental and psychological considerations into mathematical representations of conditionally rational forecasting that are compatible with the diverse ways that market participants understand and forecast outcomes. The widespread belief that rational decision-making relies solely on fundamentals, whereas irrational decisions are largely rooted in psychological considerations, is thus shown to be an artifact of determinate models.[6]

2 Formalizing an Economist's Understanding

In implementing REH or NREH, an economist formalizes his own understanding of the process underpinning aggregate outcomes in terms of a set of causal variables.

[6] In awarding the 2013 Nobel Memorial Prizes in Economics to Eugene Fama, Lars Hansen, and Robert Shiller, the Nobel Committee underscored the important insights that REH and behavioral approaches have contributed to our understanding of asset prices. However, formalized in the context of determinate models, these insights appear contradictory. NREH provides a way to reconcile these insights within the context of an internally coherent model. For an extensive discussion of how the supposed dualism between fundamental and psychological considerations has derailed macroeconomic research for decades, see Frydman and Goldberg (2011) and Frydman and Phelps (2013).

At a point in time, the structure of determinate and partly open models may share specifications of participants' preferences and other components. In simple models, this formalization results in the following semi-reduced form:

$$P_t = b_t X_t + c\left[\mathcal{F}_t^{\mathrm{M}}\left(P_{t+1}\right) - P_t\right] + \epsilon_t \tag{1}$$

where P_t represents an aggregate outcome, say a market price, the column vector X_t and row vector b_t represent a set of causal factors and their direct impacts on the price at a point in time, respectively, $\mathcal{F}_t^{\mathrm{M}}\left(P_{t+1}\right)$ denotes the market's time-t forecast of the $t+1$ price, and ϵ_t is a white noise error. To ease the presentation, we set c to a constant without altering the main conclusions of our analysis.

Determinate models represent market outcomes and causal variables with a joint probability distribution for all t. In these models, the forecast operator, $F_t^{\mathrm{M}}(\cdot)$, denotes the mathematical expectation of these outcomes, conditional on the information available at time t. Partly open model recognize that outcomes in real-world markets cannot be adequately represented with such an overarching probability distribution. In these models, $\mathcal{F}_t^{\mathrm{M}}(\cdot)$ also represents the market's forecast as conditional on the information at time t. (See Appendix A for an exposition of the properties of $\mathcal{F}_t^{\mathrm{M}}(\cdot)$ in partly open models.)

For simplicity, we characterize an economist's understanding of the X_t process as follows:

$$X_t = \mu_t + X_{t-1} + \eta_t \tag{2}$$

where the time-varying drifts in μ_t represent mean changes in the X_t variables between $t-1$ and t and η_t is a vector of white noise.[7] In determinate models, such (probabilistic) error terms are the only way in which an economist can recognize his uncertainty about outcomes. Partly open models also enable an economist to recognize non-probabilistic uncertainty concerning how the structure of the process driving outcomes might change over time.

Except for the t subscripts, the semi-reduced form in (1) has been widely used in macroeconomics and finance.[8] The micro-foundations of these models typically do

[7] In general, these drifts could depend on past values of X_t and other causal variables. For an example of such a formulation in the context of asset prices, see Frydman and Goldberg (2014b).

[8] Applications include present-value models of equity and bond prices and monetary models of the exchange rate. See, for example, Campbell and Shiller (1987) and Frenkel (1976), respectively. For micro-founded versions of these models, see Obstfeld and Rogoff (1996).

not have implications for the precise values of the parameters of their semi-reduced form. But, they often have qualitative implications—for example, that c is positive and less than one or that one or more of the parameters in b_t take on particular algebraic signs.

Formalizing the structure of an economist's model at a point in time also involves representing $\mathcal{F}_t^{\text{M}}(P_{t+1})$ in terms of a set of causal factors:

$$\mathcal{F}_t^{\text{M}}(P_{t+1}) = \phi_t V_t \tag{3}$$

where the vectors V_t and ϕ_t represent the union of causal factors that market participants consider relevant and how they interpret the impacts of these factors in forecasting P_{t+1}, respectively. As we make clear in sections 4 and 5, imposing coherence in both determinate and partly open models implies that, at a minimum, V_t consists of current and past information on the variables in X_t.[9] Internal coherence also imposes restrictions on ϕ_t, for example, that one or more parameters in this vector take on particular algebraic signs at one or more points in time.

Substituting (3) into (1), we can write an economist's understanding of the price process at a point in time in terms of causal factors with the following structure:

$$P_t = \tilde{b}_t X_t + \tilde{c}\phi_t V_t + \varepsilon_t \tag{4}$$

where $\tilde{b}_{t+1} = \frac{\tilde{b}_{t+1}}{1+c}$, $\tilde{c} = \frac{1}{!+c}$, and $\varepsilon_t = \frac{\epsilon_t}{1+c}$.

3 Partly Open versus Determinate Models

As time passes, new information on the causal factors becomes available. Moreover, the process underpinning economic outcomes can change, either because of shifts in the composition of the relevant set of causal factors or in their influence on price, or because market participants revise their forecasting strategies. In formalizing his understanding of such changes, an economist would need to allow for shifts in the $\tilde{b}_t$

[9] How much history should be included in an internally coherent model depends on how an economist represents change in the P_t and X_t processes. For example, a time-invariant REH model in which the causal variables are portrayed as random walks or AR(1) processes represents $\mathcal{F}_t^{\text{M}}(P_{t+1})$ as depending on the current values of the causal variables and their one-period mean changes. For higher order AR processes more lags would be needed.

and ϕ_t parameters. First differencing equation (4) yields:

$$\Delta P_{t+1} = \tilde{b}_t \Delta X_{t+1} + \Delta \tilde{b}_{t+1} X_{t+1} + \tilde{c}\left(\phi_t \Delta V_{t+1} + \Delta \phi_{t+1} V_{t+1}\right) + \Delta \varepsilon_{t+1} \quad \text{for all } t \quad (5)$$

which shows that price changes arise from informational effects—$\tilde{b}_t \Delta X_{t+1}$ and $\tilde{c}\phi_t \Delta V_{t+1}$—and structural change effects—$\Delta \tilde{b}_{t+1} X_{t+1}$ and $c\Delta\phi_{t+1} V_{t+1}$. Consequently, in order to generate time-series predictions from his model, an economist must constrain, *ex ante*, the structural change effects at points in time beyond $t = 0$. He must also represent any structural change effects in the X_t and V_t processes.

In formulating these constraints, an economist must decide whether to close or leave open his model's structure to unanticipated change. By doing so, an economist chooses "a world" in which he aims to represent individual decision making and its implications for aggregate outcomes.

In order to sketch the similarities and key differences between determinate and partly open accounts of change, we consider modeling structural change in a simple version of the reduced-form in (4). We assume that X_t consists of just one causal variable. We also assume that V_t consists of the single variable X_t and its trend change, μ_t, although an internally coherent, partly open representation of the market's forecast would involve other variables and their trend changes. With these assumptions we can write (4) as follows:

$$P_t = d_t X_t + g_t \mu_t + \varepsilon_t \quad (6)$$

where the X_t process is given by (2).

The vast majority of determinate models are time-invariant: they exclude at $t = 0$ all structural change by setting $d_t = d$, $g_t = g$, and $\mu_t = \mu$ for all t. When determinate models do allow for an economy's structure to change, they hypothesize at $t = 0$ that such change in all future periods can be a constrained with a probabilistic or other quantitative rule. These models specify in advance all the exact structures that could become relevant.

In order to illustrate this approach, we suppose that the structure of the price process depends on the state of the economy. Specifically, we hypothesize that there are two possible states at every point in time, $s_t = 1, 2$. We denote by $P_t^{(1)}$ and $P_t^{(2)}$

the price process in these two states, respectively:

$$P_t^{(1)} = d^{(1)} X_t + g^{(1)} \mu_t + \varepsilon_t \quad \text{or} \quad P_t^{(2)} = d^{(2)} X_t + g^{(2)} \mu_t + \varepsilon_t \quad \text{for all } t \qquad (7)$$

In order to focus on the price process, we set in advance $\mu_t = \mu$ in the X_t process for all t.

We also hypothesize, at $t = 0$, that change in the economy's structure is governed by a simple Markov switching rule: at every point in time, the model's structure can remain unchanged or switch to the other structure, with probabilities p and $(1 - p)$, respectively.[10] Consequently, this determinate model imposes, *ex ante*, two different sets of restrictions on structural change between t and $t+1$: either the parameters of the price process are constrained to remain unchanged, so that $\Delta d_{t+1} = \Delta g_{t+1} = 0$, or they are constrained to change according to the following rule:

$$\Delta d_{t+1} = \begin{cases} d^{(2)} - d^{(1)} & \text{if } d_t = d^{(1)} \\ d^{(1)} - d^{(2)} & \text{if } d_t = d^{(2)} \end{cases} \quad \text{for all } t \qquad (8)$$

$$\Delta g_{t+1} = \begin{cases} g^{(2)} - g^{(1)} & \text{if } g_t = g^{(1)} \\ g^{(1)} - g^{(2)} & \text{if } g_t = g^{(2)} \end{cases} \quad \text{for all } t \qquad (9)$$

Imposing these restrictions and using the Markov switching rule leads to the following formalization of the economist's understanding of change in the price process between any two adjacent points in time**:**

$$\Delta P_{t+1} = \begin{cases} d_t \Delta X_{t+1} + \Delta\varepsilon_t & \text{with prob } p \\ \Delta d_{t+1} X_{t+1} + d_t \Delta X_{t+1} + \Delta g_{t+1}\mu + \Delta\varepsilon_t & \text{with prob } (1-p) \end{cases} \qquad (10)$$

By contrast, IKE's partly open models formalize an economist's understanding of decision making in real-world markets in which no one can anticipate exactly in probabilistic terms when or how the process underpinning outcomes might change at any point in time, let alone in all future time periods Consequently, these models stop short of imposing determinate constraints on structural change. An IKE model

[10]For the seminal development of determinate models that make use of such a Markov switching rule, see Hamilton (1988,1994). Rather than directly specifying change between alternative structures of the reduced form of the price process, as we do here, determinate models of change typically specify switches between alternative processes for the causal variables. For example, see Mark (2009).

hypothesizes, at $t = 0$, that there are intervals of time during which unanticipated structural change can be constrained. For all points within these intervals, the model replaces the determinate constraints and the probabilistic switching rule with qualitative constraints on structural change. The model hypothesizes that these intervals are protracted, but it does not specify in advance when they may begin or end, even in probabilistic terms.

The qualitative constraints that an economist imposes depend on the context and the regularities that he thinks *ex ante* will characterize change. Our application of NREH in section 5 considers a context in which change in the price process can be characterized as moderate during protracted intervals. We define moderate structural change by constraining its impact on outcomes to be smaller in magnitude than the direct informational effect. In the context of our simple example, we have:[11]

$$|\Delta d_{t+1} X_{t+1}| < |d_t \Delta X_{t+1}| \tag{11}$$

where this constraint holds in ways that do not alter the algebraic sign of d_t across adjacent points of time.[12]

The qualitative constraint in (11) is consistent with myriad distinct structures at a point in time; consequently, it leaves the model open to unanticipated structural change from revisions of market participants' forecasting strategies and other sources at every point during an interval of time. However, the model is only partly open to such change: it hypothesizes at $t = 0$ that structural shifts will be moderate at any point in time during the intervals in which structural change is constrained. By not restricting structural change outside of the intervals of moderate change, the model is also open to revisions of strategies and other unanticipated structural change during those times. In this way, the model recognizes that an economist cannot have exact, even in probabilistic terms, of whether or how the process underpinning outcomes will change between any two points in time.

Figure 1 illustrates how an IKE model uses conditions like the one in (11) to characterize an economist's understanding of change in real-world markets. In section

[11] For applications of qualitative and contingent constraints on structural change in models of asset markets, see Frydman and Goldberg (2013d, 2014a,b) and Frydman *et al.* (2013a).

[12] In most macroeconomic contexts, the qualitative constraint in (11) implies that the sign of d_t remains unchanged at $t+1$. This is because changes in macroeconomic variables are usually smaller in magnitude than their initial levels, so that $|X_{t+1}| > |\Delta X_{t+1}|$. With the constarint in (11), this implies $|d_t| > |\Delta d_{t+1}|$.

5, we allow for both d_t and g_t to change, but for simplicity here, we set $g_t = 0$ in (6). The figure illustrates the model in an interval of time between some future t and $t+3$. At $t = 0$, the model hypothesizes that at each of these future points, d_t in (6) could be either positive or negative.

The figure illustrates the model's hypothesis that at each point in the interval $(t, t+3)$ there are two possibilities: change in the economy's structure either can or cannot be characterized as moderate. The first possibility, indicated by the "Yes" branches in Figure 1, constrains this change to be moderate. The "No" branches represent points in time at which the qualitative constraint on structural change in (11) does not hold. Both branches illustrate the key differences between determinate and partly open models.

In contrast to the quantitative constraint in (8), the qualitative constraint in (11) that holds along the Yes branches specifies neither the exact structure of P_t at the initial point in time nor the exact structure that might characterize an economist's reduced-form understanding of the price process in the succeeding period. However, if the structural change between any two adjacent points in time was moderate, the informational effect would dominate. Consequently, (11) does characterize an economist's understanding of the direction of the movements in time-series data within intervals of moderate structural change. For example, the Yes branch (A,C) illustrates how such characterizations depend on the sign of the impact of X_t on P_t: if d_t were positive (negative) at time t, ΔP_{t+1} and ΔX_{t+1} would co-move in the same (opposite) direction.

In order to account for longer-term regularities in time-series data—and thus serve as the relevant economic theory for representing conditionally rational forecasting—both determinate and partly open models must constrain in advance structural change at points in time beyond $t = 0$. As we noted above, a determinate model hypothesizes that its quantitative constraints on change over the coming period can be imposed at every point in time. The exact conditional correlations between outcomes and causal variables that are implied by these constraints formalize an economist's understanding of time series regularities from $t = 0$ to eternity.

By sharp contrast, a partly open model does not constrain structural change at every point beyond $t = 0$. The model recognizes that there are periods of time during which revisions of forecasting strategies and other unanticipated structural changes are not moderate. The "No" branches in Figure 1 illustrate such periods.

For example, the "?" at point B indicates that the model makes no prediction if the condition (11) is not satisfied. In this case, the structural change effect will dominate and regardless of the sign of d_t at point A, the model is compatible *ex ante* with both a positive or negative co-movement between ΔP_{t+1} and ΔX_{t+i}.

However, a partly open model *does* hypothesize that there are protracted intervals beyond $t = 0$ during which unanticipated structural change can be characterized with qualitative conditions, such as that in (11). During each of these intervals, the model implies qualitative regularities concerning the movements in time-series data. For example, depending on the sign of d_t at the onset of a particular interval of moderate change, ΔP_{t+1} and ΔX_{t+1} would co-move positively or negatively during the interval. The Yes branches (A,C), (C,G) and (G,O) in Figure 1 illustrate such a regularity during the interval $(t, t+3)$.

In representing conditionally rational forecasting, a key difference between partly open and determinate models is that the former leave open *ex ante* the timing of when such intervals involving a distinct qualitative regularity in the co-movements of P_t and X_t begin or end. Consequently, a partly open model makes *two* predictions *ex ante* concerning these co-movements at every point in time in an interval: it predicts that the distinct regularity either continues or ceases to be relevant. If the prevailing regularity does become irrelevant, a new interval arises during which the model may not imply any regularity in the movements and co-movements between outcomes and causal variables. The No branches (A,B), (B,D) and (D,H) illustrate such an interval between t and $t+3$.

The model hypothesizes that change eventually will again be characterized as moderate for an interval of time, which could involve the same qualitative regularity or a new one. For example, conditional on $d_{t+1} < 0$ ($d_{t+1} > 0$) at point B and structural change being moderate between $t+1$ and $t+3$, the model implies, at $t = 0$, that ΔP_{t+i} and ΔX_{t+i} will co-move negatively (positively) for $i = 1, 2, 3$.[13] Thus, in sharp contrast to their determinate counterparts, the regularities implied by a partly-open model are contingent: they characterize time-series data with qualitative regularities that last only for intervals. Moreover, because they are open to the non-moderate structural change arising from revisions of forecasting strategies and other

[13] An economist might have reasons to believe that the sign of d_{t+1} will become positive or negative. For example, in Frydman and Goldberg (2013b), we relate the sign of this parameter in a New Keynesian model of inflation to the relative size of the aggregate supply and aggregate demand effects in the macroeconomy.

sources, they do not fully specify in advance when such regularities become relevant or cease to characterize outcomes.

4 The New Rational Expectations Hypothesis

The seminal insight underpinning REH is that an economist can use his own understanding of the process driving outcomes to represent that of rational market participants. But, by selecting the class of determinate models to formalize his understanding, an REH theorist chooses to model decision-making in a hypothetical world in which participants can fully foresee when and how they will revise their forecasting strategies.

However, if we aim to represent decision-making in real-world markets, we must leave our models open to unanticipated structural change. NREH's principle of model contingency selects the class of partly open models as the relevant economic theory for representing conditionally rational forecasting by individuals who, *in pursuit of profit opportunities*, revise their understanding of the economy's structure in ways that neither they nor anyone else can fully foresee.[14]

> **Principle of Model Contingency**
>
> In order to represent how a rational individual understands and forecasts market outcomes in the pursuit of profits or other objectives, a macroeconomic model should be partly open to unanticipated structural change.

Although it bases its representations on partly open models, the new rational expectations hypothesis recognizes that internally *incoherent* models presume that market participants forego profit opportunities. NREH's second pillar—the principle of model coherence—rids partly open models of such irrationality.

4.1 The Principle of Model Coherence

We formulate the principle of model coherence in terms of five conditions, which we illustrate by showing how REH involves a particularly restrictive version of them.

[14]Popper's proposition implies that even as approximations, REH models are not useful for understanding outcomes in markets in which knowledge grows. We show in Frydman and Goldberg (2014a) that in such markets, REH models represent forecasting and decision-making by individuals who forego profit opportunities.

For ease of presentation, we make use of the time-invariant model. In order to make clear that using this model implies no loss of generality, we illustrate in Appendix B the conditions comprising the principle of model coherence in a determinate model involving probabilistic switching between exact structures, such as the one formulated in section 3. This illustration shows that, because the model fully constrains structural change, *ex ante,* it represents decision-making in a hypothetical "world" in which participants' understanding of the process driving outcomes never changes in unforeseen ways, just like its time-invariant counterpart. This illustration also prefigures section 5, where we use the principle of model coherence to represent conditionally rational forecasting on the basis of a partly open model.

Muth's insight that an economist can use his own understanding of the economy's structure to represent how the market understands and forecasts outcomes underpins both REH and NREH models. In both classes of models, once an economist assumes that a particular set of causal variables and the market's price forecast are relevant for understanding aggregate outcomes at every point in time, he should assume that profit-seeking market participants also have this qualitative understanding. He should also rely on his qualitative understanding of the process underpinning these variables in representing the market's understanding of them. We summarize these considerations with the following condition.

Condition 1 ***Causal Coherence***

> 1.1. An economist's representation of market participants' understanding of aggregate outcomes at a point in time should be compatible with his own qualitative understanding of these outcomes.
>
> 1.2. An economist should use the same functional forms to formalize his own and market participants' understanding of the causal variables that he hypothesizes are relevant.

Iterating equation (1) forward one period and isolating P_{t+1} on the left hand side yields the following expression for the economist's qualitative understanding of the price process in period $t+1$:

$$P_{t+1} = \tilde{b}_{t+1} X_{t+1} + \tilde{c} \mathcal{F}^{\mathrm{M}}_{t+1} \left(P_{t+2} \right) + \varepsilon_{t+1} \quad (12)$$

Condition 1.1 implies that this semi-reduced form can be used to represent the market's understanding of the price process at $t+1$.

In formulating REH, Muth based the market's forecasting strategy on its understanding of outcomes. NREH builds on this idea in partly open models:

$$\mathcal{F}_t^{\text{M}}\left(P_{t+1}\right)=\mathcal{F}_t^{\text{M}}\left(\tilde{b}_{t+1}X_{t+1}\right)+\tilde{c}\mathcal{F}_t^{\text{M}}\left[\mathcal{F}_{t+1}^{\text{M}}\left(P_{t+2}\right)\right] \tag{13}$$

Substituting this representation of the market's forecast of P_{t+1} into (1) yields:

$$P_t=\tilde{b}_tX_t+\tilde{c}\mathcal{F}_t^{\text{M}}\left(\tilde{b}_{t+1}X_{t+1}\right)+\tilde{c}^2\mathcal{F}_t^{\text{M}}\left[\mathcal{F}_{t+1}^{\text{M}}\left(P_{t+2}\right)\right]+\varepsilon_t \tag{14}$$

This expression formalizes an economist's understanding of the price process in terms of the market's period-t forecast of the economy's underlying structure at $t+1$, that is, $\tilde{b}_{t+1}X_{t+1}$, as well as its iterated forecasts of P_{t+2}. Because they represent decision-making in sharply different domains, REH and NREH models appeal to very different considerations in representing these iterated forecasts. But, for reasons that we make clear below, both REH and NREH represent these iterated forecasts by restricting them to be equal to the market's period-t forecast of the price at $t+2$:[15]

$$\mathcal{F}_t^{\text{M}}\left[\mathcal{F}_{t+1}^{\text{M}}\left(P_{t+2}\right)\right]=\mathcal{F}_t^{\text{M}}\left(P_{t+2}\right) \tag{15}$$

Using this representation in (14), and repeating these steps to infinity, yields:[16]

$$P_t=\tilde{b}_tX_t+\sum_{k=1}^{\infty}\tilde{c}^k\mathcal{F}_t^{\text{M}}\left(\tilde{b}_{t+k}X_{t+k}\right)+\varepsilon_t \tag{16}$$

This expression represents an economist's understanding of the price process at a point in time. According to this understanding, the price process depends on the direct effect of the causal variables—$\tilde{b}_tX_t$—as well as the market's forecast of how the structure of this effect might unfold over time. How REH and NREH represent $\mathcal{F}_t^{\text{M}}\left(\tilde{b}_{t+k}X_{t+k}\right)$ depends on how an economist formalizes his understanding of structural change in the economy. But, whether an economist represents these forecasts with a determinate or a partly open model, he ultimately relates them to

[15] With asymmetric information, the law of iterated expectations may not apply in an REH model. See, for example, Allen et al. (2006).

[16] In deriving (16), we assume that the infinite sum is convergent.

information available at time t.

REH models usually constrain the composition of the set of causal variables that the market relies on in forming its forecasts to be the same as the set that the economist considers relevant, namely current and past information available at t concerning the variables in X_t. For ease of presentation, we continue to suppose that X_t consists of only one variable.

In an internally coherent model, the amount of history concerning the casual variables that is needed in representing market participants' understanding of the price process depends on how the economist formalizes his understanding of the process underpinning the causal variables. Both determinate and partly open models formalize this understanding at a point in time in qualitative terms—for example, that μ_t in (2) is positive or negative. An economist may allow for the possibility that the market's assessment of the drift, μ_t^{M}, differs from his own, so that $\mu_t^{\text{M}} \neq \mu_t$. However, in both determinate and partly-open models, condition 1.2. constrains the functional form of the market's understanding of the X_t process to be the same as in (2):

$$X_t = \mu_t^{\text{M}} + X_{t-1} + \eta_t \tag{17}$$

where for ease of presentation we suppress the difference between the error terms in (2) and (17).

In representing how the X_t process might change and/or how market participants might revise their forecasting strategies, an economist must choose which class of models—determinate or partly open—he will rely on to formalize his understanding of market outcomes. Once an economist chooses the domain in which he aims to represent participants' decision-making, he formulates constraints on structural change that are consistent with this choice.

The following two conditions make use of Muth's idea that an economist can use his own understanding of how the economy's structure unfolds over time to formalize the market's understanding of this change. The first—domain consistency—selects the class of models—determinate or partly open—on which an economist should base his representation of the market's forecast. The second—structural consistency—selects restrictions that an economist should use to constrain structural change in his representation of the market's forecasting strategy.

Condition 2 ***Domain Consistency***

> An economist should select the same domain in which to represent his own and the market's understanding of change in the processes underpinning outcomes and the causal variables.

Although domain consistency selects the class of models that an economist should use to formalize participants' understanding of change, there are myriad models within each class. Thus, beyond choosing the class of models, an economist must select particular formalizations of how market participants understand and forecast change at a point in time. Both the REH and NREH approaches make use of the constraints on structural change that an economist uses to formalize his own understanding of change. These considerations are summarized by the following condition.

Condition 3 ***Structural Consistency***

> The constraints that an economist uses to characterize the market's understanding of change in the process underpinning outcomes and the causal variables should not conflict with those that he uses to characterize his own understanding of structural change.

In our example of section 3, the economist hypothesizes at $t = 0$ that $\mu_t = \mu$ in (2) at all future points in time. The structural consistency condition then yields the following probabilistic representation of the market's understanding of the X_t process in (17):

$$X_t = \mu^{\mathrm{M}} + X_{t-1} + \eta_t \quad \text{for all } t \tag{18}$$

Using this understanding to represent how the market forecasts the causal variable implies:

$$E_t^{\mathrm{M}}\left(X_{t+k}\right) = X_t + k\mu^{\mathrm{M}} \quad \text{for all } k > 0 \tag{19}$$

where the market forecast at each horizon k can be represented as a mathematical expectation.

In our example, we constrain the structure of the price process to be time-invariant, that is, we set $\tilde{b}_t = \tilde{b}$ at all points in time. Once an economist hypothesizes that $\tilde{b}_{t+k}X_{t+k} = \tilde{b}X_{t+k}$ at all time horizons, structural consistency also leads him to constrain the market's understanding of these effects to involve a constant impact, which we denote by β. Based on this understanding, the market's forecasts in (16) are given by:

$$\mathcal{F}_t^{\text{M}}\left(\tilde{b}_{t+k}X_{t+k}\right) = E_t^{\text{M}}\left(\tilde{b}_{t+k}X_{t+k}\right) = \beta\left(X_t + k\mu^{\text{M}}\right) \quad \text{for all } k > 0 \tag{20}$$

Using this representation in expression (16) shows that internal coherence in this determinate model formalizes the economist's understanding of the price at all t, past and future, with a single probability distribution:

$$P_t = dX_t + g\mu^{\text{M}} + \varepsilon_t \quad \text{for all } t \tag{21}$$

where $d = \tilde{b} + \tilde{c}\frac{\beta}{(1-\tilde{c})}$ and $g = \frac{\beta\tilde{c}}{(1-\tilde{c})^2}$. Applying structural consistency represents the market's understanding of the price at all t with the following overarching probability distribution:

$$P_t = \beta X_t + \gamma\mu^{\text{M}} + \varepsilon_t \quad \text{for all } t \tag{22}$$

where β and γ share the same algebraic sign as d and g, respectively. In this determinate case, structural consistency implies that only current information on X_t and an assessment of its trend change is needed to represent the information set of the market at every point in time.

In Appendix B, we show that the foregoing application of structural consistency in a time-invariant model illustrates all of the key aspects of the use of this condition in determinate models that allow for structural change. In that appendix, we consider the Markov switching example of the preceding section, which formalizes the economist's *ex ante* understanding of the price process to involve the possibility of switching between one of two exact structures at every point in time according to a Markov chain. With this formalization, structural consistency constrains at $t = 0$ the market's understanding of price process to involve as well the possibility of shifts between two exact structures according to Markov chain.

Although REH's and NREH's implications concerning movements in time-series data differ sharply, both approaches impose coherence at $t = 0$ between the model's conditional predictions and those implied by its representation of how the market forecasts outcomes and revises its forecasting strategy. The following condition imposes such coherence within a model.

Condition 4 ***The Market's Predictive Coherence***

> The model's *ex ante* predictions concerning regularities in how the causal and outcome variables move over time, and of how these regularities might change, should not conflict with those implied by its representation of the market's forecast.

In Appendix B, we show how this condition completes the determinate representation of the market's forecast in the Markov switching model of section 3. This illustration enables a sharp comparison between the quantitative predictions generated by REH models, which are completely closed to unanticipated structural change, and the qualitative predictions of NREH models, which are partly open to such change (see the next section).

In the remainder of this section, we again use the time-invariant model. This not only facilitates the exposition, but also enables us to focus on the rarely discussed yet key difference between predictive coherence on the market level and conditional rationality on the individual level. As Lucas pointed out, in order for internal coherence to rid determinate models of irrationality, an economist must constrain the forecasting strategies of every market participant to be exactly the same as that of the market in the aggregate. We will show in the next section that opening models to unanticipated structural change, as NREH does, also opens them to diversity in how rational profit-seeking market participants understand and forecast outcomes.[17]

In terms of movements of the causal variables, predictive coherence constrains at $t = 0$ the market's forecast of X_{t+1} to be consistent with the economist's forecast, that is, $E_t^{\mathrm{M}}(X_{t+1}) = E_t(X_{t+1})$. Doing so constrains

$$\mu = \mu^{\mathrm{M}} \tag{23}$$

Predictive coherence also constrains at $t = 0$ the market's forecast of next-period's price to be consistent with that of the economist:

$$E_t^{\mathrm{M}}(P_{t+1}) = E_t(P_{t+1}) \tag{24}$$

[17] As Friedrich Hayek argued in his critique of centralized planning, the key reason why markets play an essential role in modern economies is that they enable society to take advantage of the diversity of participants' knowledge. We show in Frydman and Goldberg (2014a) how a NREH present value model of stock prices provides a way to model this essenital role on the basis of conditionally rational individuals.

Based on the economist's understanding of the price process and that of the market as formalized in (21) and (22), respectively, the market's predictive coherence implies that $\beta = d$ and $\gamma = g$. Consequently, imposing internal coherence in a determinate model represents at $t = 0$ both the economist's and the market's understanding of outcomes at all t with the following overarching probability distribution:

$$P_t = dX_t + g\mu + \varepsilon_t \tag{25}$$

The market's predictive coherence, which REH imposes in a determinate model, ensures that the aggregate of market participants' predictions of how the causal variables and the price unfold over time are consistent with those of the economist. However, imposition of this condition on the aggregate level is compatible with irrationality on the part of at least some market participants. This would be the case, for example, if the model hypothesized at $t = 0$ that some individuals' predictions would invariably undershoot, and others' overshoot, the price change in such a way that, in the aggregate, the market's prediction would nonetheless be given by (55).

However, as Lucas (2001) observed, such diversity of forecasting strategies among conditionally rational individuals is ruled out in determinate models by the hypothesis that an economist's model is "relevant," in that it adequately represents the process underpinning outcomes. Under this hypothesis, representing outcomes with a single probability distribution, such as the one in (57), implies that there is only one conditionally rational way to forecast outcomes. Any individual whose forecasts systematically differed from the predictions implied by the relevant distribution would presume that he passed up profit opportunities. Hence, once an economist chooses to model decision-making in a hypothetical determinate world and imposes internal coherence, he must represent the forecasting strategy of each individual to be "essentially the same" as the one implied by the economist's model. When viewed through the prism of determinate models, it seemed that ridding macroeconomic models of irrationality on the individual level required an economist to rule out diversity in how market participants understand the economy and forecast outcomes.

However, this conclusion is an artifact of determinate models. As we show in the next section, predictive coherence on the market level in a partly open model is compatible with diversity of forecasting strategies on the part of rational individuals comprising the market. Thus, partly open models require an additional condition to ensure compatibility with rationality on the individual level.

Condition 5 ***Conditionally Rational Individual Forecasting***

> The model's *ex ante* predictions concerning the movements of the causal and outcome variables should not conflict with those implied by its representations of forecasting strategies on the individual level.

In the next section, we show how imposing internal coherence in a partly open model renders it compatible with *both* conditional rationality and diversity of forecasting strategies on the part of every individual comprising the market.

5 NREH Representations of Conditionally Rational Forecasting

NREH represents conditionally rational forecasting on the basis of partly open models. There are many ways in which an economist can open his model to revisions in how market participants understand outcomes and other sources of unanticipated structural change. However, like REH, NREH constrains the model's representation of participants' understanding and forecasting of outcomes to accord with an economist's own understanding of the process underpinning these outcomes.

In formulating his understanding of change, an economist takes into account a variety of empirical and theoretical considerations. But the restrictions that an economist would use to formalize his understanding of change are not arbitrary. As Muth and Lucas emphasized, these constraints must be "relevant" in the context in which an economist aims to represent decision-making and market outcomes. As we discussed in section 3, REH models consider a hypothetical world in which unanticipated change is irrelevant in all time periods. By contrast, NREH models consider contexts in which outcomes exhibit qualitative and contingent regularities that last for protracted stretches of time, even though unanticipated structural change occurs.

5.1 The Principle of Model Contingency

We now formalize the understanding of an economist who aims to account for outcomes in a market in which no one can fully foresee how the structure of the economy might change, and yet has reasons to hypothesize that this unanticipated change can be constrained during intervals of time with qualitative and contingent conditions. According to the principle of model coherence an economist can use this understanding to represent how market participants comprehend and forecast outcomes during these intervals.

In order to focus on the key differences between the REH and NREH approaches, we consider a partly open counterpart of the determinate model that we formulated in preceding sections. We continue to suppose that an economist's semi-reduced-form understanding of the price process and his understanding of the X_t process at a point in time are given by (1) and (2), respectively, which we recall here for convenience:

$$P_t = b_t X_t + c\left[\mathcal{F}_t^{\mathrm{M}}\left(P_{t+1}\right) - P_t\right] + \epsilon_t \tag{26}$$

$$X_t = \mu_t + X_{t-1} + \eta_t \tag{27}$$

In building his NREH model, an economist may want to recognize that the processes underpinning both the outcome and causal variables can undergo unanticipated structural changes. However, in order to facilitate a sharp comparison between the constraints that NREH and REH impose on structural change in the price process, we formulate the model's other aspects to be the same as those in the determinate model presented in sections 3 and 4.

5.1.1 A Relevant Partly Open Constraint on Change

The qualitative and contingent constraints that an economist uses to formalize his understanding of structural change are context dependent. In Frydman and Goldberg (2013b), we characterize an economist's understanding of the process underlying an economy's inflation rate at a point in time along the lines of a typical New Keynesian model. We appeal to this example here to illustrate how empirical and theoretical considerations would lead an economist to hypothesize that in many macroeconomic contexts, there are protracted intervals during which unanticipated structural change is moderate.

The economist's understanding of the macroeconomy may entail many reasons why its structure might change. In our New Keynesian example, P_t and X_t in (26) are the inflation and the real interest rate, respectively.[18] Well-known considerations have led macroeconomists to think that the direct impact of the interest rate on the inflation rate, as represented by b_t, could be either negative or positive. A negative effect would stem from typical demand-side considerations. A positive effect would arise from the supply side (for example, owing to the impact of the real interest rate on the cost of capital).[19]

This understanding of the macroeconomy implies that any factor that significantly influences either the demand side or the supply side can cause a shift in the inflation process. Consider tax policy. Firms' interest payments are deductible as expenses under current law, implying that the real cost of working capital depends on firms' marginal tax rates. A lower tax rate would lower the size of these deductions, thereby raising the cost of working capital. Shifts in tax policy, therefore, would influence the relative strength of the supply-side and demand-side effects of interest-rate changes on inflation. In fact, if the change in tax policy was large enough, it could be associated with a reversal, say, from positive to negative, in how the interest rate directly affects the inflation rate.[20]

To be sure, many other developments can influence the interest rate's impact on the inflation rate. These include changes in other supply-side factors—for example, shifts in a country's trade openness—which affect firms' ability to pass variations in marginal costs on to customers, and changes in demand-side factors, such as shifts in consumers' debt burdens, which affect households' willingness to borrow and spend at any interest rate.[21] But these and other factors tend to remain largely unchanged or change very little for protracted periods of time. During these intervals, we would expect moderate change in the inflation process between adjacent points in time, with changes on the demand or supply side not altering the algebraic sign of the

[18] The process underlying the inflation rate at a point in time is characterized along the lines of Calvo (1983) and Rotemberg (1985).

[19] See Van Wijnbegen (1983, 1985) and Neumeyer and Perri (2005) for models in which credit financing of working capital plays a key role in business-cycle fluctuations.

[20] For example, the Tax Reform Act of 1986 eliminated the investment tax credit, dramatically changed depreciation allowances, and lowered the top federal statutory tax rate for corporate income from 46% to 34% percent (the rate was increased in 1993 to its current level of 35%). Cohen *et al.* (1999) find that this tax-policy change significantly increased firms' user cost of capital.

[21] In explaining the lackluster response of private spending to a fall in interest rates in Japan in the 1990s and in the U.S. and Europe after 2008, Koo (2008, 2012) and others emphasize high debt burdens and underwater balance sheets. As with relatively large shifts in tax policy, large shifts in debt burdens can cause a reversal of which interest-rate effect—on the supply side or the demand side—is dominant.

interest rate's effect on inflation.[22]

However, major shifts in policy, institutions, and other factors do eventually occur. We would thus expect periods of moderate change in the inflation process to be punctuated by moments when this process shifted in relatively significant ways. Major shifts in the macroeconomy depend on economic, political, and financial developments that for the most part can be anticipated only dimly, if at all. Consequently, no one can fully anticipate when intervals of moderate change in the macroeconomy might begin or end.

In formalizing this partly open understanding of the underlying economy, we define moderate structural change, as in section 3, by constraining its impact on outcomes to be smaller in magnitude than the direct informational effects. Such constraints are imposed on change in the model's reduced-form structure at $t = 0$. But they are hypothesized to apply only during intervals of time, the beginning and end of which the model leaves open.

5.2 The Principle of Model Coherence

The principle of model coherence implies that, as with determinate models, an economist who adopts NREH should use his own understanding of the economy to represent how the market understands and forecasts outcomes. As in section 4, we apply this principle by imposing each of its five conditions.

5.2.1 Causal Coherence

As in the determinate case, imposing condition 1.1 on forward iterations of the semi-reduced form and assuming that the market's understanding underpins its forecasting leads to the representation in (16) for both the economist's and the market's understanding of the price process at a point in time. In both REH and NREH models, this derivation makes use of the representation of iterated forecasts in (15) at all

[22] Similar reasoning applies to potential changes in the interest-rate process. Monetary policy plays a central role, but many other financial and economic factors can influence how interest rates unfold over time. These factors include shifts in macro-prudential policy, such as changes in banks' capital requirements, and institutional changes like German reunification and the creation of the European Monetary Union. As with the inflation process, these factors tend to remain unchanged or change very little for protracted stretches of time.

horizons k:

$$\mathcal{F}_t^{\mathrm{M}}\left[\mathcal{F}_{t+k}^{\mathrm{M}}\left(P_{t+k+1}\right)\right]=\mathcal{F}_t^{\mathrm{M}}\left(P_{t+k+1}\right) \quad \text{for all } k>0 \tag{28}$$

With a determinate understanding of change, the same conditional probability distribution represents how the market forecasts P_{t+k+1} on the basis of information at t and $t+k$. Thus, the constraint that the market's iterated forecast of the price at $t+k+1$ is equal to its time-t forecast follows from the law of iterated expectations.

By opening his model to unanticipated structural change, the economist recognizes that there is no conditional probability distribution or other quantitative rule that he can apply at $t=0$ to characterize structural change in the economy. Internal coherence then leads him to hypothesize that participants comprehend at each time t that unanticipated structural change may occur and they may rely on different forecasting strategies at different horizons beyond t. Consequently, the law of iterated expectations does not apply if the economist aims to represent decision-making in markets in which no one can fully foresee how he might alter his understanding of outcomes. However, in both determinate and partly open models, the constraint (28) can be seen to follow from the constraints that both classes of models impose on diversity among market participants' forecasts.

Iterated Forecasts and Diversity A determinate model represents the forecasts by every participant i with the same conditional probability distribution as that for the market in the aggregate. In these models, the law of iterated expectations constrains iterated forecasts not only on the aggregate level, as in (28), but also on the individual level:

$$E_t^i\left[E_{t+k}^{\mathrm{M}}\left(P_{t+k+1}\right)\right]=E_t^i\left(P_{t+k}\right) \quad \text{for all } k>0 \tag{29}$$

Partly open models recognize that in real-world markets, a participant's time-t forecast of P_{t+k+1} does, in general, differ from his time-t forecast of the market's time-$t+1$ forecast of P_{t+k+1}:[23]

$$\mathcal{F}_t^i\left[\mathcal{F}_{t+k}^{\mathrm{M}}\left(P_{t+k+1}\right)\right]-\mathcal{F}_t^i\left(P_{t+k+1}\right)\neq 0 \tag{30}$$

[23] For example, in the context of stock markets, value investors like Warren Buffet look for companies whose prospects he thinks are undervalued given market prices and available information. The idea is that sooner or later the market will alter how it interprets available information and understand that they are worth more, thereby bidding up prices.

These considerations imply that in order to derive implications for time-series data, an economist must constrain not only unanticipated structural change, at least during some intervals; he must also impose restrictions on the diversity of participants' forecasting strategies. By ruling out unanticipated structural change, REH models automatically constrain a participant's time$-t$ forecast of the market's time-$t+k$ forecast of P_{t+k+1} to be exactly the same as his own time-t forecast of P_{t+k+1}.

By contrast, opening macroeconomic models to unanticipated structural change also opens them to the diversity of participants' forecasting strategies. As Hayek (1945) argued, this diversity cannot be fully comprehended by anyone—a premise that implies that an economist can anticipate neither how a profit-seeking participant forecasts future outcomes, nor how the market will forecast them.

REH models constrain $F_t^i \left[\mathcal{F}_{t+k}^{\text{M}} \left(P_{t+k+1}\right)\right] - F_t^i \left(P_{t+k+1}\right) = 0$, at $t = 0$. Partly open models impose a much weaker constraint at $t = 0$ on this difference. These models recognize that an economist cannot know *ex ante* what this difference might be for any individual at any time horizon. Moreover, they recognize that he has no reason to suppose *ex ante* that the market's period-t forecast of outcomes at time $t+k+1$—$\mathcal{F}_t^{\text{M}} \left(P_{t+k+1}\right)$—will be higher or lower than its period-t prediction of its forecast of outcomes at that horizon based on the information available at time $t+k$—$\mathcal{F}_t^{\text{M}} \left[\mathcal{F}_{t+k}^{\text{M}} \left(P_{t+k+1}\right)\right]$ Consequently, we constrain the aggregate of the difference in (30) as follows,

$$\sum_i \left\{\mathcal{F}_t^i \left[\mathcal{F}_{t+k}^{\text{M}} \left(P_{t+k+1}\right)\right] - \mathcal{F}_t^i \left(P_{t+k+1}\right)\right\} = \mathcal{F}_t^{\text{M}} \left[\mathcal{F}_{t+k}^{\text{M}} \left(P_{t+k+1}\right)\right] - \mathcal{F}_t^{\text{M}} \left(P_{t+k+1}\right) = 0 \quad \text{for all } k > 0$$

This constraint imposes *no* restrictions on the market's forecasting strategy across time horizons. Moreover, in contrast to REH models, this representation of iterated forecasts on the market level is compatible with diversity of participants' forecasting strategies and information, as well as with partly open models' core premise that outcomes in real-world markets cannot be adequately represented with an overarching probability distribution.

5.2.2 Domain Consistency

Domain consistency implies that the economist, having chosen a partly open model to formalize his understanding of change in the price process, should also represent market participants' understanding and forecasting to be partly open. In general, he will recognize that the process driving both the outcome and causal variables may undergo unanticipated structural changes. However, as would be the case in a determinate model, he must impose constraints on structural change *ex ante*, at time $t = 0$, in order to derive time-series implications from his model.

5.2.3 Structural Consistency

Unlike with REH, imposing structural consistency in a partly open model enables an economist to recognize that the union of information sets used by market participants in forecasting outcomes not only includes current and past information on the casual variables in X_t, but is also much larger than his own. It also enables him to recognize that different sets of variables and factors may be relevant for representing the market's forecasts of the $\tilde{b}_{t+k}X_{t+k}$ terms in (16) at different horizons. For example, in the context of the present-value model of equity prices, X_t includes dividends, the future values of which depend on companies' profitability and prospects. In forecasting dividends over the short term, one might rely largely on recent trends in company earnings and overall economic activity, whereas at longer horizons, company reports and the composition of management teams may be more relevant.

These considerations lead us to represent each forecast term in (16) as follows:

$$\mathcal{F}_t^{\text{M}}\left(\tilde{b}_{t+k}X_{t+k}\right) = \rho_{t,k}X_t + \lambda_{t,k}\mu_t^{\text{M}} + \sigma_{t,k}Z_{t,k}$$

where the vector $Z_{t,k}$ represents horizon-specific additional factors and the parameters in $\rho_{t,k}$, $\lambda_{t,k}$, and $\sigma_{t,k}$ represent the weights that the market attaches to these factors. With these representations, we can express the economist's understanding of the price process at a point in time as

$$P_t = \tilde{b}_tX_t + \sum_{k=1}^{\infty}\tilde{c}^k\left(\rho_{t,k}X_t + \gamma_{t,k}\mu_t^{\text{M}} + \sigma_{t,k}Z_{t,k}\right) + \varepsilon_t \tag{31}$$

where we note that there may be common variables and factors in the specific $Z_{t,k}$'s.

Here, we abstract from the complication of additional casual variables in the market's information set and constrain V_t to include only the variables in X_t and their trend changes.[24] In order to highlight the main differences between REH and NREH, we suppose that the economist formalizes his understanding of the X_t process to be time-invariant and sets $\mu_t = \mu$ in (27). We also continue to suppose that X_t involves a single variable. With these simplifications, the economist's qualitative understanding of the price process at a point in time becomes,

$$P_t = d_t X_t + g_t \mu^{\text{M}} + \varepsilon_t \tag{32}$$

where $d_t = \tilde{b}_t + \rho_t$, $\rho_t = \sum_{k=1}^{\infty} \tilde{c}^k \rho_{t,k}$, and $g_t = \sum_{k=1}^{\infty} \tilde{c}^k \gamma_{t,k}$ and the model's microfoundations typically have implications for the algebraic signs that d_t and γ_t may take at one or more points in time.

Causal coherence then implies the following representation for the market's understanding of the price process:

$$P_t = \beta_t X_t + \gamma_t \mu^{\text{M}} + \varepsilon_t \tag{33}$$

where the parameters d_t, β_t and γ_t in this NREH model recognize that the economist will need different structures to represent the price process and conditionally rational forecasting at different points in time. Rational market participants use their understanding to forecast outcomes. Using (27) and (33), we can express the market's forecast of the change in price at a point in time as follows:

$$\mathcal{F}_t^{\text{M}}\left(\Delta P_{t+1}\right) = \beta_t \mu^{\text{M}} + \mathcal{F}_t^{\text{M}}\left(\Delta \beta_{t+1} X_{t+1}\right) + \mathcal{F}_t^{\text{M}}\left(\Delta \gamma_{t+1}\right) \mu^{\text{M}} \tag{34}$$

Substituting this representation into (26) implies the following expression for the economist's reduced-form understanding of the change in the market price between

[24] However, the reduced form in (32) does implicitly allow for the influence of the market's additional variables to the extent that they are correlated with X_t. The parameters d_t and γ_t in (32) depend on these correlations. For NREH models that explicitly recognize the importance of the market's additional informational variables, see Frydman and Goldberg (2014a,b).

any two adjacent points in time:

$$\Delta P_{t+1} = \tilde{b}_t \Delta X_{t+1} + \Delta \tilde{b}_{t+1} X_{t+1} + \tilde{c}\left[\Delta\beta_{t+1}\mu^{\text{M}} + \Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\beta_{t+2}X_{t+2}\right) + \Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\gamma_{t+2}\right)\mu^{\text{M}}\right] + \Delta\varepsilon_{t+1} \quad (35)$$

where $\Delta\mathcal{F}^{\text{M}}_{t+1}(\cdot)$ denotes, for example, $\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\beta_{t+2}X_{t+2}\right) - \mathcal{F}^{\text{M}}_{t}\left(\Delta\beta_{t+1}X_{t+1}\right)$. The first term on the right-hand side of this expression—$\tilde{b}_t \Delta X_{t+1}$—is a direct informational effect. All the other terms involve structural change effects: $\Delta\tilde{b}_{t+1}X_{t+1}$ and $\Delta\beta_{t+1}\mu^{\text{M}}$ represent change in the underlying economy and revisions in how the market interprets movements of X_t in forecasting outcomes across adjacent periods, respectively, whereas $\Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\beta_{t+2}X_{t+2}\right)$ and $\Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\gamma_{t+2}\right)\mu^{\text{M}}$ represent revisions in the market's forecasts of structural change. The conditional forecast implied by the model can now be expressed as:

$$\mathcal{F}_t\left(\Delta P_{t+1}|\tilde{b}_t, \beta_t, X_t\right) = \tilde{b}_t\mu + \mathcal{F}_t\left(\Delta\tilde{b}_{t+1}X_{t+1}\right) + \tilde{c}\mathcal{F}_t\left[\Delta\beta_{t+1}\mu^{\text{M}} + \Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\beta_{t+2}X_{t+2}\right) + \Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\gamma_{t+2}\right)\mu^{\text{M}}\right] - \varepsilon_t$$

Constraining Structural Change In order to derive time-series implications from the model, an economist must constrain the structural-change effects in his expression of the conditional forecast at $t = 0$. As we discussed in section 5.1.1, his understanding leads him to constrain these effects only during intervals of moderate change. He cannot not know *ex ante* when these intervals might begin or end. However, he hypothesizes that such intervals will occur and, when they do, they tend to be protracted. We denote the beginning and end of interval j by $T_j + 1$ and T_{j+1}, respectively.

Our definition of moderate change constrains structural change in every interval as follows:[25]

$$\left|\tilde{b}_t\mu\right| > \left|\mathcal{F}_t\left(\Delta b_{t+1}X_{t+1}\right)\right| + c\left|\mathcal{F}_t\left[\Delta\beta_{t+1}\mu^{\text{M}} + \Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\beta_{t+2}X_{t+2}\right) + \Delta\mathcal{F}^{\text{M}}_{t+1}\left(\Delta\gamma_{t+2}\right)\mu^{\text{M}}\right]\right|$$

$$sgn\left(\tilde{b}_t\right) = sgn\left(\tilde{b}_{t-1}\right) \quad \text{and} \quad sgn\left(\beta_t\right) = sgn\left(\beta_{t-1}\right) \tag{36}$$

where $|\cdot|$ denotes an absolute value. The first qualitative and contingent condition constrains structural change in the underlying economy and revisions in the market's forecasting strategy to be moderate. The second condition implies that the structural changes are sufficiently moderate so that the algebraic signs of the informational effects in the economist's understanding of the underlying economy as well as his representation of the market's forecasting strategy remain unchanged during the intervals of moderate change.

Imposing these conditions at $t = 0$, like in section 2, yields qualitative and contingent predictions concerning the movements of the outcome and casual variables during the intervals of moderate change:

$$\mathcal{F}_t\left[sgn\left(\Delta P_{t+1}\right)\right] = sgn\left(\tilde{b}_t\mu\right) \tag{37}$$

$$\mathcal{F}_t\left[sgn\left(\Delta P_{t+1}\Delta X_{t+1}\right)\right] = sgn\left(\tilde{b}_t\right) \tag{38}$$

for t between $T_j + 1$ and T_{j+1} and all j. These predictions involve constraining ε_t *ex ante*. This error term represents influences on the price process during intervals of moderate change that the economist does not explicitly specify either in quantitative or qualitative terms. REH models impose the strong assumption that what the economist does not know *ex ante* is exactly the same as what the market does not know, that is, they set $\varepsilon_t = \varepsilon^{\text{M}}_t$ at every point in time, past, present, and future. An NREH model imposes a weaker constraint. It recognizes that an economist cannot know in advance whether ε_t will take on a positive or negative value. Consequently, he constrains this error term to equal its average of zero in deriving *ex ante* qualitative predictions from his model.

Structural consistency implies that the economist relies on his own understanding

[25] There are other ways to formalize moderate structural change in the model. The key is to do so in a way that implies time series regularities.

of change in the price process to represent how market participants understand and forecast change. He thus formalizes the market's understanding as partly open using constraints on change that do not conflict with his own formalization in (36). In doing so, he constrains revisions in the markets' forecasting strategy to be moderate during the intervals of time in which change in the underlying economy is also moderate. In effect, the economist hypothesizes at $t = 0$ that market participants in the aggregate will understand when the underlying economy undergoes moderate change and thus the market will revise its forecasting strategy in moderate ways during these periods.[26]

In order to yield qualitative predictions concerning the market's forecast of movements in the outcome and causal variables, the economist must constrain its forecast of the structural-change effects in his representation in (34). Structural consistency constrains these effects at $t = 0$ to be smaller in magnitude than the direct informational effect at each point in time during the intervals of moderate change, that is:

$$|\beta_t \mu^{\mathrm{M}}| > \left| \mathcal{F}_t^{\mathrm{M}} \left(\Delta\beta_{t+1} X_{t+1} \right) + \mathcal{F}_t^{\mathrm{M}} \left(\Delta\gamma_{t+1} \right) \mu^{\mathrm{M}} \right| \tag{39}$$

for t between $T_j + 1$ and T_{j+1} and all j. With this constraint, the representation in in (34) implies the following qualitative and contingent predictions:

$$\mathcal{F}_t^{\mathrm{M}} \left[sgn \left(\Delta P_{t+1} \right) \right] = sgn \left(\beta_t \mu^{\mathrm{M}} \right) \tag{40}$$

$$\mathcal{F}_t^{\mathrm{M}} \left[sgn \left(\Delta P_{t+1} \Delta X_{t+1} \right) \right] = sgn \left(\beta_t \right) \tag{41}$$

for t between $T_j + 1$ and T_{j+1} and all j.

The IKE constraints on change in (36) and (39) are consistent with myriad possible structures at each point in time during intervals of moderate change. However, imposing these constraints nonetheless implies qualitative and contingent predictions, because, conditional on a structure at t, they imply that informational effects dominate structural-change effects.

[26] In Frydman and Goldberg (2014a), we point out that this supposition is an implication of the premise that market participants' aggregate understanding of change encompasses that of the economist. REH models embody a particularly restrictive form of this premise: they presume that an economist's understanding is "essentially the same," in quantitative terms, as that of the market.

5.2.4 The Market's Predictive Coherence

The IKE model in equations (35), (36) and (39) predicts, at $t = 0$, that movements in the outcome and causal variables will be characterized by distinct qualitative regularities during protracted time intervals, depending on the algebraic sign of $\tilde{b}_t$ in those intervals. The model specifies in advance neither the timing of these intervals' beginning or end nor what the sign of $\tilde{b}_t$ will be in any particular interval. It also yields no time-series predictions outside these intervals.

Imposing predictive coherence at the aggregate level in this IKE model ensures that the market's conditional forecasts at every point in time during intervals of moderate change do not conflict with the model's qualitative time-series predictions during these periods, that is, with those in (40) and (41). Predictive coherence, therefore, implies that

$$sgn\left(\beta_t\right) = sgn\left(\tilde{b}_t\right) \quad \text{and} \quad sgn\left(\mu^{\mathrm{M}}\right) = sgn\left(\mu\right) \tag{42}$$

for t between $T_{j+1} - T_j + 1$ and all j.[27]

As in our determinate example, predictive coherence entails using the economist's understanding to represent that of the market. However, in an IKE model, this understanding is qualitative. As such, the economist uses only the algebraic signs of the parameters of his own formalization in representing the market's forecast. By doing so, he also hypothesizes at $t = 0$ that market participants—in the aggregate—will understand when the underlying economy undergoes moderate change, and thus that they will revise their forecasting strategy in moderate ways during these periods.

5.2.5 Conditionally Rational Individual Forecasting

The market's predictive coherence leads an economist to hypothesize at $t = 0$ that, in the aggregate, market participants' forecasts of the movements in the outcome and causal variable are qualitatively consistent with his model's predictions during the intervals of moderate change in the economy's structure. However, a key feature of NREH models is that they do not restrict all conditionally rational participants to forecast the same qualitative time-series regularities as those implied by the model.

[27] We have formalized the economist's understanding of the X_t process as determinate. Consequently, predictive coherence also implies that $\mu^{\mathrm{M}} = \mu$.

This diversity arises because the model does not specify in advance when intervals of moderate structural change will begin or end. At each point in time, the model implies two predictions: either structural change will be moderate or it will not. Consequently, conditional on this understanding, it is rational for a market participant to forecast either one of these possibilities. The individual rationality condition implies that if a participant forecasts a continuation of moderate structural change at any moment in the interval, he should forecast movements in the data that are consistent with the qualitative regularities predicted by the economist and the market in (37) and (38).[28] However, if at any moment he forecasts that structural change will cease to be moderate, he may forecast a change that is either the same as or different from the qualitative regularities in the data—or no regularity at all.[29]

The key to modeling diversity of forecasting strategies among conditionally rational participants is that the economist himself recognizes that change in the economy's structure is contingent. By doing so, his model yields myriad conditionally rational ways to understand and forecast outcomes. Moreover, because the model implies two qualitative predictions concerning structural change at every point in time, it accounts for participants' forecasts of time-series regularities that are qualitatively consistent with that of the market and others that are not. If, by contrast, an economist formalizes his understanding of change with a determinate model, he hypothesizes that there is a single joint probability distribution governing market outcomes and thus only one conditionally rational way to think about and forecast those outcomes.

Rationality of Bulls and Bears Determinate models are particularly problematic in accounting for asset-price fluctuations on the basis of conditionally rational decision-making. These prices have a tendency to undergo swings of irregular duration and magnitude away from and back toward benchmark values. As an asset-price swing unfolds, say rising above the benchmark, market participants must forecast whether it will continue or end and usher in a price downswing (or no swing at all).

[28] In general, it would take some time once an interval of moderate structural change began for market participants to observe the qualitative time-series regularities in the interval. We assume here that they can detect these regularities quickly; if they forecast moderate structural change between t and $t+1$, the sign of their individual β_t's is the same as that of the economist's b_t.

[29] This individual rationality condition is not necessarily satisfied. An individual who forecasts moderate change in the economy's structure and nonetheless changes the sign of the weight that he attaches to X_t would be conditionally irrational.

At each point in time, some participants forecast that the upswing will continue (the bulls), while others forecast that it will end (the bears). The interplay between bulls and bears in asset markets leads to an enormous volume of trading.[30] However, attempting to account for such behavior on the basis of a determinate model typically leads economists to hypothesize that either the bulls or the bears, or both, systematically forego profit opportunities.[31]

By contrast, our NREH model of asset-price swings and risk accounts for the presence of both bulls and bears without presuming that either group is irrational (Frydman and Goldberg, 2014b). By leaving the model partly open to unanticipated change, the model implies two predictions: at every point in time, a price swing either continues or it does not. It is thus conditionally rational for some participants to forecast that the swing will continue and for others to forecast that it will end. Moreover, the model's contingent representation of change, together with NREH's imposed internal coherence, enables it to explain the irregular nature of asset-price swings.

6 Concluding Remarks

Since the early 1970s, macroeconomists have come to rely on models that exclude, *ex ante*, unanticipated changes in how market participants make decisions and how aggregate outcomes unfold over time. Once determinate models are upheld as the relevant economic theory, REH as a representation of conditionally rational forecasting follows on logical grounds.

The choice to stick with determinate models has had a profound impact on the direction of macroeconomic research over the last four decades. The emergence of behavioral finance is a case in point. Behavioral economists have uncovered much evidence of the empirical failure of REH models. Yet, puzzlingly, they have not jettisoned the core belief that underpins much of contemporary macroeconomics: determinate models are the relevant theory for understanding outcomes, and imposing REH in such models provides the way to represent rational forecasting. Conse-

[30] For example, in its 2013 survey, the Bank for International Settlements estimates that the volume of foreign exchange traded every day in currency markets is more than $5 trillion. See www.bis.org/publ/rpfx13.htm.

[31] For such models of irrationality, see Brunnermeier (2001) and references therein.

quently, behavioral economists formalize their findings of how participants behave with internally incoherent determinate models, which presume that individuals forego profit opportunities endlessly. According to this approach, the empirical failure of REH models stems from decision-making by market participants who are not smart enough or rational enough to forecast according to an economist's determinate model (Barberis and Thaler, 2003).[32]

The arguments advanced in this paper point to a very different interpretation of behavioral economists' findings: determinate models are not the relevant economic theory for understanding outcomes in real-world markets; thus, REH does not represent conditionally rational forecasting in these markets. We have pointed out that REH models represent decision-making by participants in real-world markets who forego profit opportunities endlessly. It is not surprising, therefore, that reliance on these models has led to many so-called puzzles in macroeconomics, especially in financial markets.[33]

We have proposed partly open models as the relevant economic theory for understanding outcomes in real world markets and NREH as a way to select from that class of models those that are compatible with rational decision-making. These models explore the possibility that, although no one can fully foresee how change in the economy's structure will unfold, there are protracted intervals of time during which this change can be characterized *ex ante* with qualitative conditions. We have shown that during these intervals, a NREH model implies distinct qualitative regularities in the movements of outcomes and causal variables. NREH models recognize that when these regularities begin or end cannot be fully specified in advance with a probabilistic rule.

A key question is how to confront NREH's qualitative and contingent implications

[32] NREH indicates that the usual dualism concerning the importance of fundamentals, on which REH theory has focused, and that of psychological considerations, which behavioral economists have emphasized, is largely an artifact of determinate models. As we have argued in Frydman and Goldberg (2011), the importance of unanticipated structural change implies that both fundamental and psychological considerations play a role in representing how profit-seeking rational participants understand the economy and make decisions. Indeed, this point goes back to Keynes (1936, p. 136). For a discussion of how NREH representations of conditionally rational forecasting are open to the influence of psychological considerations, see Frydman and Goldberg (2014a,b) and Frydman *et al.* (2014c,d).

[33] In other words, Lucas's (2001, p.13) compelling argument – that "if your theory reveals profit opportunities, you have the wrong theory",– goes a long way toward explaining the empirical difficulties encountered by REH models.

with time-series data. In our empirical research on asset markets, we have explored the possibility that the qualitative and contingent regularities implied by IKE models can be approximated *ex post* with a piece-wise linear econometric model. To leave the econometric analysis partly open, we rely on procedures that do not specify in advance the timing of or the ways in which relationships in the data might shift across linear pieces. Our results indicate that partly open models are crucial for resolving many of the anomalies that financial economists have encountered using determinate models.[34]

Appendix A
The Forecast Operator

In order to illustrate the properties of the forecast operator, we consider the following process relating an outcome, denoted by Y_t, in terms of two causal variables, Z_t^1 and Z_t^2:

$$Y_t = a_t^1 Z_t^1 + a_t^2 Z_t^2 \tag{43}$$

where a_t^1, a_t^2 are parameters. Let $\mathcal{F}_t\left(Y_{t+1}\right)$ denote the forecast of Y_{t+1} based on the realizations of Z_t^1 and Z_t^2 and the understanding in **(43)** Iterating one period and using the forecast operator yields:

$$\mathcal{F}_t\left(Y_{t+1}\right) = \mathcal{F}_t\left(a_t^1 Z_t^1 + a_t^2 Z_t^2 + a_t^1 \Delta Z_{t+1}^1 + a_t^2 \Delta Z_{t+1}^2 + \Delta a_{t+1}^1 Z_t^1 + \Delta a_{t+1}^2 Z_t^2\right) \tag{44}$$

where $\Delta Z_{t+1}^i = Z_{t+1}^i - Z_t^i$, and $\Delta a_{t+1}^i = a_{t+1}^i - a_t^i$ for $i = 1, 2$.

In partly open models, some or all of these changes do not have a probabilistic representation. Consequently, the forecast operator represents a point forecast stem-

[34] Frydman and Goldberg (2007, chapter 13), Kozlova (2013), and Frydman *et al.* (2014b) show that allowing for unanticipated structural change helps to resolve the forward-discount anomaly in currency markets. Frydman *et al.* (2014b,e) find that a NREH model of the risk premium, which relates risk not to the volatility of returns, but to departures of the asset price from benchmark values, can account for excess returns in currency markets and the pattern of forward-rate biasedness across developed and developing countries, respectively. Frydman and Goldberg (2007, chapter 15) and Sullivan (2013) find that allowing for unanticipated structural change is crucial to resolving the Meese and Rogoff (1983) exchange-rate-disconnect puzzle. Frydman and Goldberg (2014b) and Frydman *et a*l. (2014a) show how a NREH model of asset-price swings can account for the persistence of currency fluctuations. Frydman *et al.* (2014c) shows that, once we allow for structural change, the present value model of stock prices is broadly consistent with empirical evidence. Frydman *et al.* (2014d) tests a NREH present-value model of stock prices, one that explicitly incorporates fundamental, psychological, and social considerations, and finds temporary co-integrating relationships among all three types of factors.

ming from the model's qualitative and contingent formalization of the economist's or market participants' understanding of the economy's structure. Although partly open model's qualitative constraints are compatible with myriad forecasting strategies at a point in time, for each of them the forecast operator in partly open models shares the following properties with a mathematical conditional expectation:

1. The time-t forecast of the sum is the sum of the forecasts:

$$\mathcal{F}_t\left(Y_{t+1}\right) = \mathcal{F}_t(a_t^1 Z_t^1) + \mathcal{F}_t\left(a_t^2 Z_t^2\right) + \mathcal{F}_t\left(a_t^1 \Delta Z_{t+1}^1\right) + \mathcal{F}_t\left(a_t^2 \Delta Z_{t+1}^2\right) + \mathcal{F}_t\left(\Delta a_{t+1}^1 Z_t^1\right) + \mathcal{F}_t\left(\Delta a_{t+1}^2 Z_t^2\right) \tag{45}$$

2. The time-t forecast of the parameters or variables that are included in the conditioning set at t is equal to these parameters or variables:

$$\mathcal{F}_t(a_t^i Z_t^i) = a_t^i Z_t^i \quad \text{for } i = 1, 2 \tag{46}$$

$$\mathcal{F}_t\left(a_t^i \Delta Z_{t+1}^i\right) = a_t^i \mathcal{F}_t\left(\Delta Z_{t+1}^i\right) \text{ and } \mathcal{F}_t\left(\Delta a_{t+1}^i Z_t^i\right) = Z_t^i \mathcal{F}_t \Delta a_{t+1}^i) \quad \text{for } i = 1, 2$$

A partly open model does not necessarily represent structural change in all of its components in non-probabilistic terms. For those components that are represented probabilistically, the forecast operator is simply the mathematical conditional expectation based on the probability distribution characterizing those components.

Appendix B

Internal Coherence in a Markov Switching Model

We illustrate the application of the conditions of structural consistency and the market's predictive coherence in the determinate model of section 3, which makes use of the representations in (21) and (22).

Structural Consistency

In that example, the economist hypothesizes at $t = 0$ that the price process can switch between one of two exact structures according to a Markov chain. He formalizes his understanding of this change with the determinate constraints in (8) and (9). Structural consistency implies that he uses this understanding of change to represent that of the market:

$$\begin{array}{l} s_t = 1 : P_t^{(1)} = \alpha^{(1)} + \beta^{(1)} X_t + \varepsilon_t \ \text{ or} \\ s_t = 2 : P_t^{(2)} = \alpha^{(1)} + \beta^{(2)} X_t + \varepsilon_t \end{array} \qquad \text{for all } t \tag{47}$$

according to a simple Markov chain,

$$P_{t+1} = \begin{cases} P_{t+1}^{(1)} & \text{with prob } p^{\mathrm{M}} \text{ if } s_t = 1 \text{ and } (1-p^{\mathrm{M}}) \text{ if } s_t = 2 \\ P_{t+1}^{(2)} & \text{with prob } p^{\mathrm{M}} \text{ if } s_t = 2 \text{ and } (1-p^{\mathrm{M}}) \text{ if } s_t = 1 \end{cases} \quad \text{for all } t \tag{48}$$

where $\alpha^{(i)} = \gamma^{(i)}\mu^{\mathrm{M}}$, $i = 1, 2$.

Having imposed a probabilistic rule on structural change, we can again represent the market's time-t forecast of next period's price with a mathematical expectation in the two states:

$$E_t^{\mathrm{M}}\left[P_{t+1}|s_t = 1, X_t\right] = p^{\mathrm{M}}\left[\alpha^{(1)} + \beta^{(1)}\left(X_t + \mu^{\mathrm{M}}\right)\right] + \left(1-p^{\mathrm{M}}\right)\left[\alpha^{(2)} + \beta^{(2)}\left(X_t + \mu^{\mathrm{M}}\right)\right] \tag{49}$$

$$E_t^{\mathrm{M}}\left[P_{t+1}|s_t = 2, X_t\right] = \left(1-p^{\mathrm{M}}\right)\left[\alpha^{(1)} + \beta^{(1)}\left(X_t + \mu^{\mathrm{M}}\right)\right] + p^{\mathrm{M}}\left[\alpha^{(2)} + \beta^{(2)}\left(X_t + \mu^{\mathrm{M}}\right)\right] \tag{50}$$

where the intercepts in the two states are $\alpha^{(i)} = \gamma^{(i)}\mu^{\mathrm{M}}$, $i = 1, 2$.

Without a loss of generality, we assume $s_t = 1$. This yields the following reduced-form understanding of the price process at time t:

$$P_t^{(1)} = \frac{1}{1+c}\tilde{b}^{(1)}X_t + \frac{c}{1+c}\left[\alpha^{(12)} + \beta^{(12)}\left(X_t + \mu^{\mathrm{M}}\right)\right] + \varepsilon_t \tag{51}$$

where $\alpha^{(12)} = p^{\mathrm{M}}\alpha^{(1)} + \left(1-p^{\mathrm{M}}\right)\alpha^{(2)}$ and $\beta^{(12)} = p^{\mathrm{M}}\beta^{(1)} + \left(1-p^{\mathrm{M}}\right)\beta^{(2)}$. At time $t+1$, the price process is in one of the following two states:

$$P_{t+1}^{(1)} = \frac{1}{1+c}\tilde{b}^{(1)}X_{t+1} + \frac{c}{1+c}\left[\alpha^{(12)} + \beta^{(12)}\left(X_{t+1} + \mu^{\mathrm{M}}\right)\right] + \varepsilon_{t+1} \tag{52}$$

$$P_{t+1}^{(2)} = \frac{1}{1+c}\tilde{b}^{(2)}X_{t+1} + \frac{c}{1+c}\left[\alpha^{(21)} + \beta^{(21)}\left(X_{t+1} + \mu^{\mathrm{M}}\right)\right] + \varepsilon_{t+1} \tag{53}$$

where $\alpha^{(21)} = \left(1-p^{\mathrm{M}}\right)\alpha^{(1)} + p^{\mathrm{M}}\alpha^{(2)}$ and $\beta^{(21)} = \left(1-p^{\mathrm{M}}\right)\beta^{(1)} + p^{\mathrm{M}}\beta^{(2)}$.[35]

The expressions in (51)-(53) involve the parameters characterizing the market's understanding: $\beta^{(1)}, \beta^{(2)}, \gamma^{(1)}, \gamma^{(2)}, \mu^{\mathrm{M}}$ and p^{M}. REH determines the values of these parameters by supposing that, at time $t = 0$, the market's forecasts in (49)-(50) are consistent with the model's predictions. Such internally coherent models imply, at

[35] The bracketted terms in both (52) and (53) involve an expectation concerning $t+2$. Hence, (53), which assumes the process is in state 2 at $t+1$, involves the parameters $\alpha^{(21)}$ and $\beta^{(21)}$ rather than $\alpha^{(12)}$ and $\beta^{(12)}$.

$t = 0$, quantitative predictions concerning the movements and co-movements of time series data between all adjacent points of time in the future.

The Market's Predictive Coherence

In terms of the causal variable's movements, predictive coherence constrains, at $t = 0$, the market's forecast of X_{t+1} to be consistent the economist's forecast, that is, $E_t^{\mathrm{M}}[X_{t+1}|X_t] = E_t[X_{t+1}|, X_t]$. Doing so constrains

$$\mu = \mu^{\mathrm{M}} \tag{54}$$

Predictive coherence also constrains, at $t = 0$, the market's forecast of next-period's price to be consistent with that of the economist:

$$E_t^{\mathrm{M}}[P_{t+1}|s_t = 1, X_t] = E_t[P_{t+1}|s_t = 1, X_t] \tag{55}$$

The model's expectation of P_{t+1} is given by

$$E_t[P_{t+1}|s_t = 1, X_t] = pP_{t+1}^{(1)} + (1-p)P_{t+1}^{(2)} \tag{56}$$

where $P_{t+1}^{(1)}$ and $P_{t+1}^{(2)}$ are give in (52) and (53). Using (54) and equating the constant terms and parameters of X_t on both sides of (55) constrains, at $t = 0$, the parameters of the market's forecasting strategy:

$$\beta^{(1)} = \tilde{b}^{(1)}, \beta^{(2)} = \tilde{b}^{(2)}, \gamma^{(1)} = \tilde{c}\tilde{b}^{(1)}, \gamma^{(2)} = \tilde{c}\tilde{b}^{(2)}, p^{\mathrm{M}} = p \tag{57}$$

These quantitative constraints, in turn, imply that

$$E_t^{\mathrm{M}}[P_{t+1}|s_t = 1, X_t] = A + \left[p\tilde{b}^{(1)} + (1-p)\,\tilde{b}^{(2)}\right] X_t \tag{58}$$

which when substituted into the semi-reduced form in (1) yields the reduced-form understanding,

$$P_t^{(1)} = \tilde{c}A + \left[\tilde{b}^{(1)} + p\tilde{b}^{(1)} + (1-p)\,\tilde{b}^{(2)}\right] X_t + \varepsilon_t \quad \text{for all } t \tag{59}$$

where $A = \left[p\left(\tilde{c}\tilde{b}^{(1)} + \tilde{b}^{(1)}\right) + (1-p)\left(\tilde{c}\tilde{b}^{(2)} + \tilde{b}^{(2)}\right)\right]\mu$. The economist's reduced-form representation of of the price process when $s_t = 2$ is derived in similar fashion.

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