Learning, Expectations, and the Financial Instability Hypothesis

Martin Guzman\textsuperscript{1} and Peter Howitt\textsuperscript{2}

Working Paper No. 33

December 2015

ABSTRACT

This paper analyzes what assumptions on formation of expectations are consistent with Minsky’s Financial Instability Hypothesis (FIH) and its corollaries. The FIH establishes that financial relations evolve over time turning a stable system into an unstable one. Financial crises would be more likely to occur, and more severe if they occur, the longer the previous crisis recedes into the past. We show that the hypothesis is consistent with assumptions on formation of expectations that imply learning from realization of states and inconsistent with the assumption of full information rational expectations.

JEL codes: D84, E32, F34, G01

Key words: Expectations, Endogenous Financial Fragility, Financial Crises

\textsuperscript{1} Columbia University GSB. E-mail: mg3463@columbia.edu

\textsuperscript{2} Brown University, Department of Economics, and NBER. E-mail: Peter Howitt@brown.edu
We are grateful to Daniel Heymann, Axel Leijonhufvud, and Joseph Stiglitz for useful comments and discussions. Martin Guzman is thankful to the Institute for New Economic Thinking for support.
1 Introduction

Expectations matter. Many economic and financial decisions depend on the perception of future incomes and prices. The evolution of expectations, and how correct they are over time, determines the stability of the system.

Waves of optimism generally lead to increases in spending and borrowing. The intertemporal consistency of those decisions depends on the fit between expectations and realizations. Large discrepancies between those objects may severely affect the capacity for fulfilling promises (as reflected in the original contracts), as the agents may not have the resources to honor their obligations. Broken promises in a large scale are the defining feature of financial crises (Heymann, 2009).

It has largely been recognized that how agents form expectations affects the system’s behavior—from Keynes’ animal spirits (Keynes, 1936) to Minsky’s Financial Instability Hypothesis (FIH) (Minsky, 1975, 1986, 1992).

Minsky’s FIH, also described by Kindleberger (1978) and revived during the last US financial crisis, is a theory of the impact of debt on system behavior that also incorporates the manner in which debt is validated. It draws upon the credit view of money and finance developed by Schumpeter (1934). One of its corollaries is that over periods of prolonged prosperity, an economy transits from financial relations that make for a stable system to financial relations that make for an unstable system. This dynamic is characterized by a build-up of leverage. Hence, the more prolonged the period of prosperity, the higher the likelihood of a financial crisis, and the more severe the crisis if it occurs. Financial stability would lead to a stronger perception that crises are a thing from the past.

There is a challenge in constructing a persuasive theory of endogenous financial instability. If one assumes that expectations are simply arbitrarily given, then a sudden change in the perceived probability distribution that governs expectations can obviously give rise to marked changes in economic decisions and the state of the economy. The problem with that theory is that the task of explaining instability is too easy. This is a legitimate critique of “animal spirits”.

But a theory of financial instability can emerge with no such arbitrary assumptions on expectations. This paper shows that more refined theories for formation of expectations, that assume that distributions of beliefs change over time as a
function of the new information the economy receives, are consistent with the FIH. On the other hand, it is impossible to reconcile the FIH with the assumption of full information rational expectations (FIRE) – as it is challenging to reconcile the observed evolution of beliefs over time with any theory of rational expectations (Gluzmann, Guzman, and Howitt, 2014).

Under FIRE, the realization of states are uninformative of future events. All that needs to be known about the distributions that govern the evolution of the economy is already known by definition. Therefore, the absence of crises in the recent past would not affect the expected probabilities of a future crisis.

Under assumptions on expectations formation that allow for learning, agents will update beliefs over time based on what they observe. Long periods of stability will lead to the perception that the economy is permanently more stable. It is well known that for any utility function that implies precautionary savings, a greater variance of expected permanent income will lead to lower consumption and more savings in the future, and vice versa. Hence, a perception of a permanently more stable economy would generally lead to more borrowing, making the economy more vulnerable to the realization of bad states.

Our analysis focuses on the interaction between expectations formation, news, and perceived volatility (as reflected in the volatility of agents’s expectations). We assume a process for output that features permanent and transitory shocks. Agents attempt to identify what type of shock the economy is receiving in every period in order to form correct expectations about future output growth. We introduce a measure for the volatility of expectations that reflects how expectations change over time as a response to the signals the economy receives. We show how this measure of volatility is related to the assumptions on expectations formation, shedding light on how learning may affect the dynamics of the system.

The rest of the article is organized as follows. In section 2 we introduce a process for output growth and define a measure of volatility of expectations about output growth, whose evolution will be our main object of interest. In section 3 we analyze how different assumptions for formation of beliefs affect the perceived stability of the system, and we study the consistency of those results with the hypothesis of endogenous financial instability. Section 4 concludes. Importantly, this paper lays out the theoretical foundations for an empirical analysis of Minsky’s
FIH that we perform in a related paper (Gluzmann, Guzman, and Howitt, 2014).

2 Volatility of output growth expectations

2.1 A process for output growth

Assume that the growth rate of output at time $t$, $g^y_t$ is given by

$$g^y_t = g_t + z_t - z_{t-1}$$  \hspace{1cm} (1)

where $g$ and $z$ represent permanent (cumulative) and transitory shocks, respectively.\(^1\)

Suppose that transitory shocks $z_t$ follow an AR(1) process,

$$z_t = \rho z_{t-1} + \epsilon^z_t$$  \hspace{1cm} (2)

with $|\rho_z| \in (0,1)$, $\epsilon^z_t \sim N(0, \sigma^z_2)$, where $\rho_z$ and $\sigma^z_2$ represent the persistence and the variance of the transitory shocks, respectively.

Also, suppose that permanent shocks $g_t$ are described by

$$g_t = (1 - \rho_g)\mu_g + \rho_g g_{t-1} + \epsilon^g_t$$  \hspace{1cm} (3)

with $|\rho_g| \in (0,1)$, $\epsilon^g_t \sim N(0, \sigma^g_2)$, where $\mu_g$ is the steady-state growth rate of output, and $\rho_g$ and $\sigma^g_2$ represent the persistence and the variance of the permanent shocks, respectively.

2.2 A measure of volatility of expectations

Definition 1 – Change in expectations. $CE_{t-1,t}$ is the change in output growth expectations from period $t-1$ to $t$,

$$CE_{t-1,t} = |E_t g^y_{t+1} - E_{t-1} g^y_t|$$

\(^1\)The growth rate for output is derived from an output process $y_t = e^{\Gamma_t}$, with $\Gamma_t = e^{\theta_t} \Gamma_{t-1}$, where $y_t$ denotes output in period $t$. 

For the output process defined above, 

\[ CE_{t-1,t} = |(1 - \rho_g/t) \mu_{g/t} - (1 - \rho_g/t) \mu_{g/t-1} + \rho_g/t \tilde{g}_t - \rho_g/t-1 \tilde{g}_{t-1} + (\rho_z/t - 1) \tilde{z}_t - (\rho_z/t-1 - 1) \tilde{z}_{t-1}| \]  

(4)

where \( \rho_{i/t} \) is belief on the persistence of shocks of type \( i, i = g, z \) and \( \mu_{g/t} \) is the belief on the steady-state growth rate of output, all conditional on the available information in period \( t \). There are four sources of changes in expectations: changes in the belief about the steady-state growth of output, changes in the beliefs about the persistence parameters, changes in the belief about the permanent shock, and changes in the belief about the transitory shock.

**Definition 2 – Volatility of expectations.** \( VOE(t_0, T) \) is a measure of the stability of expectations between periods \( t_0 \) and \( T \):

\[ VOE(t_0, T) = \frac{1}{T - t_0} \sum_{t = t_0}^{T} CE_{t-1,t} \]

We can think of period \( t_0 \) as the period in which the last financial crisis occurred. For the purposes of analyzing Minsky’s FIH, we are interested in knowing how the stability (or volatility) of expectations evolves since the last financial crisis. A larger value of \( VOE \) means a higher volatility of expectations.

We define the change in \( VOE \) between periods \( T \) and \( T + 1 \) as

\[ \Delta VOE_{T,T+1} \equiv VOE(t_0, T + 1) - VOE(t_0, T) \]  

(5)

Then,

\[ \Delta VOE_{T,T+1} = \frac{1}{T + 1 - t_0} [CE_{T,T+1} - VOE(t_0, T)] \]  

(6)

With forward-looking agents, the past does not matter *per se*, but only to the extent that it affects expectations about future variables. However, for the purposes of empirical analysis, a measure of volatility of expectations that is calculated using past data may be useful. This will be the case if we are interested in analyzing retrospectively how a dimension of financial crises, as their severity, is related to the volatility of expectations in the periods between consecutive crises.
(as we do in Gluzmann, Guzman, and Howitt, 2014). In a full model with consumption decisions, optimizing forward-looking agents would be interested in the perceived variance of output growth.

3 Formation of Beliefs and Volatility of Expectations

This section distinguishes three different assumptions on formation of beliefs, namely full information rational expectations (FIRE), Bayesian learning, and non-Bayesian learning, and analyzes the interaction between realization of states and perceptions of stability (or volatility) as a function of those assumptions.

**Definition 3 Great Moderation** We define a period of Great Moderation as $T$ consecutive similar “good” growth shocks $g_t^y$, such that

$$g_{t+j}^y = g_{t+j-1}^y + \theta_{t+j}$$

(7)

$$\theta_{t+j} = (1 - \rho_\theta)\mu_\theta + \rho_\theta \theta_{t+j-1} + \epsilon_t^\theta$$

(8)

with $\epsilon_t^\theta \in B_c(\mu_\theta)$, $\epsilon > 0$, $\text{Var}(\epsilon_t^\theta) \leq \sigma^2 \epsilon < \sigma^2 y/t$, and $\mu_\theta > \mu_{y/t}$, for $T$ sufficiently large.

We define a period of Great Moderation as a period of relatively low output growth volatility and high average output growth.\(^2\)

3.1 Full Information Rational Expectations (FIRE)

As in Aguiar and Gopinath (2007), under FIRE agents can perfectly identify what share of the aggregate shock is transitory or permanent.

Under FIRE,

$$g_{t+1}^y = E_t g_{t+1}^y + \epsilon_{t+1}^y$$

(9)

where

$$\epsilon_{t+1}^y = \epsilon_{t+1}^y + \epsilon_{t+1}^z$$

(10)

\(^2\)“Good” signals refer to high growth, not to the information content of signals.
with
\[ E(e^y_{t+1}) = 0 \quad \& \quad E[e^y_t \cdot e^y_{t+1}] = 0 \quad \forall t > 0 \]
which is ensured by \( E[e^y_t \cdot e^y_{t+1} = 0], E[e^z_t \cdot e^z_{t+1} = 0], E[e^y_t \cdot e^z_{t+1} = 0] \), and \( E[e^z_t \cdot e^y_{t+1}] = 0 \).

That is, the actual growth rate of output should be equal to the expected growth rate plus a forecast error that should have a sample mean equal to zero and should have no serial autocorrelation under the null of FIRE.

By definition, forecast errors provide no useful information about the future. More generally, neither the past nor the present provides any useful information for inferring the true parameters that govern the productivity shocks, which are perfectly known by the agents.

**Definition 4** Variance of output growth:
\[
\text{Var}(g^y_t) \equiv \sigma^2_y = \frac{\sigma^2_{g/t}}{1 - \rho^2_{g/t}} + \frac{\sigma^2_{z/t}}{(\rho_{z/t} - 1)^2} \quad (11)
\]

The following proposition shows that under FIRE more stability would not lead to a perception of changing stability in the system.

**Proposition 1** Under FIRE, a period of Great Moderation does not affect the perceived volatility of the system.

**Proof 1** By definition of FIRE, \( \rho_{z/t} = \rho_z, \sigma^2_{z/t} = \sigma^2_z, \rho_{g/t} = \rho_g, \) and \( \sigma_{g/t} = \sigma^2_g \).

Hence, \( \sigma^2_{y,t} = \sigma^2_y \forall t \). QED

Under FIRE, then, more macroeconomic stability would not lead agents to perceive that the system is less risky, a result that is incompatible with Minsky’s FIH. The intuition is simple: under FIRE there is nothing to learn, and a stream of consecutive similar signals will not lead to any change in beliefs.

As under FIRE \( \rho_{i/t} = \rho_i \) and \( \mu_{g/t} = \mu_g \forall t \), the expression for \( CE_{t-1,t} \) becomes
\[
CE_{t-1,t} = |\rho_g(g_t - g_{t-1}) + (\rho_z - 1)(z_t - z_{t-1})| \quad (12)
\]

Output growth shocks will affect \( CE \) hence also \( VOE \), but under FIRE a measure that uses past data is irrelevant for the agents.
3.2 Bayesian learning

Suppose that at time \( t \) agents observe the aggregate shock \( g_t^y \) but they do not observe its composition. The best they can do is to use past information (that was used to form beliefs on the parameters of the distributions of shocks) and the signal they receive (i.e. the aggregate shock), in order to infer what share of the shock is permanent and what is transitory.

Assuming normality for the distribution of errors, the optimal strategy to decompose the aggregate shock will be to use a linear estimator, that is, a Kalman filter that results in posterior beliefs according to

\[
at_t = k_1a_{t/t-1} + k_2g_t^y
\]

where \( a_t = E(\alpha_t/I_t) = \begin{bmatrix} z_t & z_{t-1} & g_t \end{bmatrix}' \), \( \alpha_t = \begin{bmatrix} z_t & z_{t-1} & g_t \end{bmatrix}' \), and \( k_1 \) and \( k_2 \) are the Kalman coefficients that determine the mapping of prior beliefs \( a_{t/t-1} \) and signals into posterior beliefs of transitory and permanent components of the aggregate shock. The Kalman coefficients depend on the parameters that govern the productivity processes \( g_t \) and \( z_t \).

We can write

\[
\alpha_t = T\alpha_{t-1} + c + R\eta_t
\]

where

\[
T = \begin{bmatrix} \rho_z & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho_y \end{bmatrix}; c = \begin{bmatrix} 0 \\ 0 \\ (1 - \rho_y)\mu_y \end{bmatrix}; R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; \eta_t = \begin{bmatrix} \epsilon_t \\ \epsilon_t^g \end{bmatrix}
\]

with \( \eta_t \sim N(0, Q) \), \( Q = \begin{bmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \).

The Kalman filters are

\[
k_1 = I - PZ'(ZPZ')^{-1}Z
\]

\[
k_2 = PZ'(ZPZ')^{-1}
\]

where \( P \) is the steady-state covariance matrix of estimation errors \( P_t = E[(\alpha_t - \)
\(a_t(a_t - a_t')\), calculated following the Riccati equation as:

\[
P = TPT' - TPZ'PZ'\!^{-1}ZPT' + RQR
\]  

(17)

The prior belief is given by

\[
a_{t/t-1} = Ta_{t-1} + c
\]  

(18)

From the updating process of the Kalman coefficients, we obtain the following result:

**Result 1** The share of \(g_t'\) attributed to \(\tilde{g}_t\) is increasing in \(\sigma^2_{g/t}\). 

In the Bayesian context, the parameters that govern the productivity processes are recursively updated when a new signal arrives. The updating process also features the following result:\(^3\)

**Result 2** If \(\frac{\tilde{g}_t - \mu_{g}}{g_t} > \frac{\tilde{z}_t}{g_t}\), then \(\frac{\sigma^2_{g/t}}{\sigma^2_{z/t}} > \frac{\sigma^2_{g/t-1}}{\sigma^2_{z/t-1}}\)

Result 2 establishes that when the part of the aggregate shock that is attributed to the permanent component is larger than the one attributed to the transitory shock, the perceived relative variance of the permanent component will increase. 

A period of Great Moderation will lead agents to believe that output volatility is low, and to believe that deviations from \(\mu_{g/t}\) are mostly transitory. As a consequence, it will decrease the volatility of expectations. The next proposition summarizes this result.

**Proposition 2** Under unbiased Bayesian learning, a period of Great Moderation decreases \(\text{VOE}\).

**Proof 2** From (4), \(CE\) is decreasing in the variability of the parameters that govern the distribution of shocks. Under Great Moderation, \(\rho_{g/t} \rightarrow \rho_{g}^{GM}\), \(\rho_{z/t} \rightarrow \rho_{z}^{GM}\), \(\sigma_{g/t}^2 \rightarrow \sigma_{g,GM}^2\), and \(\sigma_{z/t}^2 \rightarrow \sigma_{z,GM}^2\), where \(\rho_{g}^{GM}\), \(\rho_{z}^{GM}\), \(\sigma_{g,GM}^2\), and \(\sigma_{z,GM}^2\) are

\(^3\)Note that with Bayesian learning, past beliefs on the transitory shock \(\tilde{z}_{t-1}\) are also updated.
the parameters with the best fit to the distribution of shocks for the period $[t, t+T]$.

QED

For agents that learn, a long period of stability with high growth will suggest that the economy is permanently on a superior steady state with permanently lower volatility. In a model in which agents maximize expected utility, the decrease in perceived volatility would lead to less precautionary savings (provided the utility function implies precautionary savings), and the perception of higher growth would lead to more borrowing, a result that is consistent with Minsky’s FIH.

### 3.3 Stochastic-gain learning

Suppose that agents either do not know the processes that govern productivity, or that they know them but do not use that information in order to forecast future output growth.\(^4\) Suppose that they follow a simple rule, called stochastic-gain learning (SGL): If forecast errors are small, the individual adjusts her expectations by using a decreasing gain parameter; if forecast errors are large, the individual suspects that there was a change of regime and uses a constant gain parameter, which assigns more importance to information from the present. This algorithm is introduced in the literature by Sargent (1993), and further explored by Marcet and Nicolini (2003) and Milani (2007). It is non-Bayesian learning, as agents may not use all the information they have optimally, but it satisfies a set of “desirable” conditions for a learning process, in the sense of assuming minimum deviations from rationality (see below).

Let $g_t^p$ be the growth rate of output at time $t$ and let $E_t$ denote the expectation over variables at time $t$. Analytically, SGL is represented by

$$E_t g_{t+1}^p = E_{t-1} g_t^p + \kappa (g_t^p - E_{t-1} g_t^p)$$

\(^4\)We can think of situations in which agents have information since a very distant past, but they think not all of that information is representative of the current productivity process.
with

\[
\kappa_t = \begin{cases} 
\frac{1}{t} & \text{if } \frac{1}{S} \sum_{s=0}^{S} (|y_{t-s} - E_{t-s-1}y_{t-s}|) < v_t^y \\
\kappa & \text{if } \frac{1}{S} \sum_{s=0}^{S} (|y_{t-s} - E_{t-s-1}y_{t-s}|) \geq v_t^y
\end{cases}
\] (20)

where \( \kappa_t \) is the gain parameter that determines how expectations respond to forecast errors, \( S \) is the relevant time horizon for comparing recent forecast errors with historical forecast errors, and \( v_t^y \) is the mean absolute deviation of historical forecast errors, which is recursively updated. When the agent switches back to a decreasing-gain parameter, the parameter is reset to \( \frac{1}{\kappa^{-1} + t} \), with \( t = 1 \) after the switch.\(^5\)

SGL satisfies desirable lower bounds on rationality (introduced by Sargent (1993), proved in Marcet and Nicolini (2003)). Let \( p^{\varepsilon,T} \) be the probability that the perceived errors in a sample of \( T \) periods will be within \( \varepsilon > 0 \) of the rational expectations error. Then, SGL satisfies AR, EDR, and IC:

**Definition 5** Asymptotic rationality (AR): \( p^{\varepsilon,T} \) converges to 1 for \( T \) large, \( \forall \varepsilon > 0 \).

**Definition 6** Epsilon-Delta Rationality (EDR): for \( (\varepsilon, \delta, T) \), \( p^{\varepsilon,T} \geq 1 - \delta \), for \( \delta > 0 \).

**Definition 7** Internal consistency (IC): After \( T \) periods, the average perceived error using the rule for \( \kappa_t \) is smaller than under any alternative learning rule for \( \kappa_t \) (studied only for “moderately high” \( T \)).

AR implies asymptotic good forecasts, while EDR and IC imply good forecasts along the transition.

With SGL, we have

\[
CE_{t-1,t} = \kappa_t |g_t^y - E_{t-1}g_t^y|
\] (21)

From the definition of \( \kappa_t \), we infer that a more stable economy, in which average forecast errors are smaller, is associated with a smaller average \( \kappa_t \). Therefore,

\(^5\)Note that under SGL, the expectation on output growth can also be written as a convex combination of the aggregate signal and the prior belief: \( E_t g_{t+1}^y = \kappa_t g_t^y + (1 - \kappa_t) E_{t-1}g_t^y \).
under SGL stability will also lead to lower volatility of expectations. Proposition 3 summarizes this result.

**Proposition 3** Under SGL, a period of Great Moderation decreases $\text{VOE}$.

**Proof 3** From (21), $\text{CE}$ is decreasing in $\kappa_t$. From (20), under Great Moderation average $\kappa_t$ will decrease over time. Hence, average $\text{CE}$ will decrease over time, implying the proposition. QED

4 Conclusions

Modeling endogenous financial fragility and understanding its determinants remain as key issues in macroeconomics. There have been some progress over these themes in recent years\(^6\), as there was much progress in older times as well,\(^7\) but there is still much to learn about them.

This paper focused on Minsky’s Financial Instability Hypothesis, and analyzed the consistency of different commonly used assumptions on expectations formation with such hypothesis.

We firstly showed that under the full information rational expectations (FIRE) hypothesis, that mechanism is not valid. The intuition is simple: under FIRE there is nothing to learn, and a stream of consecutive similar signals says nothing about the perceptions of stability for the future. We then showed how this mechanism is valid under hypotheses that contemplate learning, either in a Bayesian or non-Bayesian fashion. In models with learning, changes in agents’ output growth expectations are smaller when they believe that observed changes in output growth are mostly of a transitory nature, or when forecast errors are smaller, which is more likely in a more stable economy. Furthermore, the smaller updates reinforce the general perception that the share of output variance that is due to transitory shocks has increased, which in turn leads to even smaller updates of forecasts, i.e. to more stability of expectations.


But if the perceptions on the stability of the system turn out to be wrong, instability may emerge. A testable hypothesis, consistent with an environment in which agents update beliefs over time on the basis of learning from realization of states, is that the resulting instability will tend to be more severe when the period of previous stability—not representative of the real stability of the system, as in our definition of Great Moderation—lasted longer. Our paper, besides clarifying what assumptions are consistent with the emergence of endogenous financial instability, has also laid out the foundations for an empirical analysis of this hypothesis.

References


