(Second Draft)

Marshall on Market Adjustment

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Once a dominant figure in the field, Alfred Marshall=s reputation declined dramatically in the second half of the twentieth century and his intellectual influence almost ceased altogether. Much maligned by some modern theorists, most notably perhaps by Paul Samuelson (1967/1972), he more commonly suffered the total neglect of others (e.g., Arrow & Hahn 1971). This widespread negative appraisal is the result of a common misinterpretation. Marshall has been judged by the standard of a later, and to him conceptually alien, neoclassicism and in this way found wanting.

The conceptual tools of Book V of Marshall=s **Principles** survived for decades in applied price theory and found a refuge in undergraduate instruction even longer. But even this apparatus is more well-known than well understood.

Marshall did not build from choice theory, did not represent decisions as solutions to constrained optimization problems, and made no strong assumptions about the >rationality= of agents. He saw the Aordinary business of life@ as the interaction in real time of innumerable adaptive processes. In his time, the mathematical tools for dealing with the dynamics of such complex systems were lacking. Marshall managed, however, to construct a brand of *static theory* that allowed a fairly disciplined analysis of a host of questions pertaining to the complex dynamical system of a highly developed economy.

To understand his approach, one should start from a simple observation: Marshall drew his supply-and-demand diagrams with quantity on the horizontal and price on the vertical axis. In contrast, Walras had price on the horizontal, quantity on the vertical axis. Both, of course, obeyed mathematical convention and nothing better illustrates our confusions over neoclassical economics than the universal habit of drawing Walrasian schedules in Marshallian space. Unlike, Walras, Marshall did not start from quantities chosen as functions of price, but from valuations as functions of quantities.

Supply-price and demand-price schedules are not loci of optimal points. A supply-price is the

minimum price a producer would accept in order to continue producing the corresponding quantity. Any higher price would spell abnormal profit. Similarly, a demand-price is the *maximum* price the consumer will be willing to pay. Any price lower than $p^d(q)$ will obviously be preferred. It would imply consumer's surplus.

The conceptual experiments underlying $p^s(q)$ and $p^d(q)$ schedules are thus quite different from those generating the usual $q^s(p)$ and $q^d(p)$ functions. Marshall's schedules are upper and lower boundaries of sets. Consequently, it is in general not legitimate to treat "quantity-into-price" functions as inverses of "price-into-quantity functions" (Hicks, 1956, Ch. IX) as has frequently been done in the textbook literature. The confusions surrounding the Marshallian demand curve is in no small measure due to failure to respect this distinction.

Adaptive behavior

If points on $p^s(q)$ and $p^d(q)$ do not indicate a correspondence between price and most preferred quantity, what might be the analytical use of these constructs? The answer is that they provide the rules of routine adaptation to changes in the market environment; they reflect "the forces that cause movement (**Principles**, Preface). For the consumer, the rule is: If demand-price exceeds the market price, increase purchases; in the opposite case, cut back. For the producer: If supply price exceeds the market price, reduce output; in the opposite case, expand.

Marshall's construction of the consumer's demand-price schedule is instructive. He postulated additive cardinal utility and a constant marginal utility of money. The marginal utility of the n-th ounce of tea divided by the marginal utility of a shilling has the dimension of number of shillings per ounce tea, i.e., the demand price for that n-th ounce. Marshall was aware that his assumption of a constant marginal utility of money could not be strictly true and might not even be a good approximation for a good with large weight in the household=s consumption basket. But mathematical inexactitude bought him a healthy dose of behavioral versimilitude. The Marshallian consumer who knows the value of money in terms of utility to himself, need not know all prices and need not solve an n-dimensional Lagrangean before making any purchase as must his Slutsky cousin. He can go *shopping,* making sequential decisions of what to buy and not to buy depending on whether his demand-price for a good exceeds or falls short of the market price. This consumer may explore an environment that he does not completely know beforehand (Heymann & Leijonhufvud

1995, pp. 156-60). Marshall's treatment of consumer demand is thus a good example of how his theory lends itself to analysis of economic *processes* in a manner of which Walrasian theory is entirely incapable.

In what follows, the behavior routines embedded in the demand-price and supply-price schedules will be referred to as Marshall's *"laws of motion"* to emphasize that we are dealing with agents adapting in real time. We may think of the agents as gradient climbers. This would be consistent with Marshall's repetitive insistence on continuity: *Natura non facit saltum.* But one would have to suppose that, in environments with which they have had considerable past experience, agents would be more likely to "jump" to the near neighborhood of their new (local) optima.

The Market Process Simplified

To complete the supply-and-demand model, the process of price formation has to be defined. This is not just a matter of postulating a "law of motion" for the price. Process models require a description of the institutions of the market that enable the orderly interaction of producers and consumers. It will be convenient to start with a "stylized" model which, while it is not literally to be found in Marshall, will serve to delineate for a contemporary audience the main conceptual and methodological differences between Marshall and the present-day theory descended from Walras.

Suppose we are dealing with the market for fish in a port city that is the base for a fishing fleet. Fish is assumed not to be storable (as in Marshall's time, before the days of frozen fish). So we do not need to bother about inventories carried from one market day to the next. The boats, *j* in number, go out each night and return in the early morning with their catch. The fleet's entire catch, $q_T = \sum_j q_{Tj}$, is brought to the local auction house. (As in medieval times, "forestalling" is prohibited).

Assume an auction process that will find the market clearing price for the aggregate catch of day T. The analytically simplest case would be a *tâtonnement*:

$$(1) \quad p_{t+1} = f[D(p_t) - q_T] + p_t = f[x_t] + p_t$$

where D(p) is the demand schedule which assumes that demanders have moved to equate their demand prices with the market price and where f (.) aggregates all the relevant market information into a well-defined excess demand. The adjustment function, f (.), is assumed to have the standard properties: f (0) = 0; f (x > 0) > 0 and f (x < 0) < 0.

The iterative process in p_t is assumed to converge within the day, T, to the market clearing

price, p_T^* . This point attractor is Marshall's market day equilibrium.

The individual firm (the *j*-th boat owner) compares the price thus realized with the marginal cost incurred (his supply price) in order to decide whether to expand or contract. To bring in a larger catch the boat will need to set more nets during the night and perhaps add men to the crew.

$$(2) \quad q_{T+1,j} = h_j [s_j (q_{T,j}) - p^*_T] + q_{T,j}$$

Denoting the excess supply price, π, the adjustment function should have the properties: $h[0] = 0$; $h[\pi > 0] < 0$; and $h[\pi < 0] > 0$. The boats go out again to return with q_{T+1} which is again auctioned off at the price, p^*_{T+1} . The iteration in q_T will, it is assumed, settle down to the point attractor, $q^*_{T+\tau}$ which is Marshall's short-period equilibrium.

This market process "finds" the equilibrium without relying on an aggregate $q^s(p)$ supply schedule. The short-period equilibrium defines a single point on Marshall's industry supply schedule. To generate the rest of it, one has to imagine letting the market process find the point attractor associated with each possible position of the demand schedule.

A representative firm?

A Walrasian model is constructed in three consecutive stages: (i) individual choices of optimal quantities, (ii) aggregation and (iii) equilibrium, with the equilibrium defined as the mutual *consistency* of all individual plans. In contrast, the model above first finds the short-period equilibrium, defined as the *constancy* (zero rate of change) of industry output. At this point, the theorist has two options about how to connect this equilibrium of aggregate output to underlying individual firm behavior.

One is to postulate a firm that is "representative" in the sense that it has no tendency to change its output when the industry is in equilibrium. For the competitive firm, this would be the case when marginal cost incurred equals the price received. The correspondence between the "state of rest" of the industry and that of the firm need hold only for the representative firm. In Marshall, firms are not all of the same size or age, nor do they have the same technical knowledge or cost structures or the same responsiveness to changes in their environment. We need not require all agents to be at rest in this version of short period industry equilibrium. It would be true only "on average." But, in this conception, the correspondence between market price and the rate of output at which the industry would temporarily be "at rest" could hardly be assumed to be single-valued.

We would have to recognized a narrow *range* of possible equilibria for the same underlying demand and cost conditions. The theory becomes inherently stochastic. One should note also that, in this kind of construction, the *identity* of the representative firm cannot be counted on to stay the same over time. The time-path of industry output, therefore, cannot be represented as the time-consistent optimal trajectory of *the* representative firm

However, while Marshall made use of a representative firm in his theory of long-run equilibria, he did not introduce it in his analysis of the short period. The second alternative, then, would be to insist that all firms had completed their output adjustment in short-period equilibrium. This equilibrium concept is more stringent but at the cost of making it far less useful. Adjustments take place in real time and an equilibrium concept that "waits" for the slowest laggard will never be exactly applicable unless the frequency of shocks is very low indeed. Marshall was also apologetic about his equilibrium – it was a moving target, never actually attained. To a present-day econometrician this creates an obvious problem, namely, that the relationship between theoretical and observable quantities is, strictly speaking, unknown.

The equality of marginal cost and price which characterizes the equilibrium of the firm is familiar from the literature on optimal economic behavior. But the two should not be confused. In Marshall, the optimality condition is to be interpreted in *ex post* rather than *ex ante* terms. Walrasian prices and quantities are properly interpreted as expected or planned, respectively, not as realized (although the ever-present equilibrium assumption will of course conflate the two). Feedback-governed behavior (as in eq. 2 above) is backward-looking. Current behavior is adjusted in the light of outcomes. Marshall's quantities are realized and, if the representative firm conception advocated above is adopted, they would in principle be observable by an outsider analyst and provide data for the econometrician.

Marshall's Market

The stylized model above serves to make clear the conceptual differences between Marshallian and Walrasian modes of construction. But in simplifying, it also exaggerates. Marshall was never so simple or clearcut and he became ever less so with increasing age and added editions of the **Principles** . A few notes will have to suffice:

(1) Marshall did not have the market day price determined through a *tâtonnement*. In the

one instance where he discusses the problem in some detail, namely his country town corn market example (Bk V, Ch. II), trading takes place as a series of transactions between pairs of farmers and dealers. In such a double auction, the prices in successive transactions ordinarily differ somewhat. Marshall recognized this but minimized the problem by the assumptions of the particular case he chose to consider. His corn market is concentrated in space and in time with a limited number of participants, where everyone is sufficiently well informed "to prevent him from taking a lower price or paying a higher price than others..." (**Principles**, p. 284). If some transactions were made on a less well-informed basis, he argued, the distributive wealth effects would be insignificant so that the market could still be expected to settle down to a price not significantly different from the intersection of his demand- and supply-price schedules. It is worth noting that the evidence from the many double auction experiments carried out so many decades after the last edition of the *Principles* provide much support for Marshall's case (Friedman and Rust, 1993).

(2) Our stylized model is faithful to Marshall in some important respects which, however, do little credit to either. Marshall proposes that the process leading to the short-period equilibrium evolves as a sequence of daily corn-market equilibria. Fair enough, *except* he wants to apply his *static method* to the market day and short-period equilibria. Price, and in the short run output, is to be determined by the intersection of static demand and supply functions. To make this at all plausible, he sketches a picture of a market that is concentrated, and indeed isolated, in both space and time. The notion of *market clearing* is well-defined when the carrying over of inventories and intertemporal substitution is neglected. The setting is in fact as artificial as that of the typical double auction laboratory experiment. The theory falls short of dealing with an ongoing market as Hicks (1965, p. 52) has noted: "Whenever there is a possibility of *substitution over time* ... the self-containedness of the short period will break down." In dealing with an ongoing market, the supply-equal-demand condition "cannot be used to determine price, in Walras's or Marshall's manner" (Hicks, 1989, p. 11).

(3) The demand-price and supply-price schedules discussed in Books III and V are derived from underlying steady-state properties of utility and production functions. But clearly these are not the demand and supply functions representing the relevant propensities to transact on successive market days in Marshall's corn or fish market. The demand in those markets is exercised not by consumers but by dealers. The dealer's demand is not derived from the steady-state utility functions

of consumers. The simple 'laws of motion' discussed above might suggest that the market would work even with (near-) "zero intelligence" traders (Gode & Sunder 1993). But the dealer must be in some measure a speculator. To survive in his trade, he must have *learned*, for example, that the housewives who are the ultimate purchasers will readily rearrange their menus for the week in response to rather small variations in the price of fish. So his market day demand-price will be far less elastic (or, alternatively, his demand more price elastic) than the steady-state demand-price schedules of consumers. The equilibrating process is thus far less "mechanical" than the 'laws of motion' might suggest.

It is of some interest to note that Marshall has the dealer's demand-price for corn depending (inversely) on his marginal utility of money just as in the example of the consumer above. But the marginal utility of money in this application obviously has no connection to the agent's marginal utility (in consumption) of income. It is rather an expression for what Keynes would later term "liquidity preference." But Marshall provides no theory of liquidity preference to support his use of the marginal utility of money concept in the market process analysis.

(3) Hicks has also several times stressed another limitation of the theory, namely, that it resupposes a market form that probably was largely disappearing already in Marshall's own time (Hicks, 1965, Ch. V:5, 1989, Ch. I) . In Marshall's corn-market, prices are determined through daily "higgling and bargaining" between producers and middlemen. In twentieth century markets for brand-name manufactured goods produced under conditions of decreasing cost, the producers set prices and manage production and inventory scheduling accordingly. The simple 'laws of motion' discussed above will not suffice to deal with this market form. Marshall recognized that these markets were growing in importance but he did not formulate an adaptive routine that would fit such a market and drive it to a stable short-period equilibrium.

(4) Just as dealers will be conservative in adjusting their demand-price in the market, the producers will not adjust output to day-by-day oscillations in the price of fish. The high frequencies are filtered out and the low frequencies ignored in the adjustments toward the short-period equilibrium. Producers constantly adjust production but not as mechanically backward-looking as in the stylized example. Expectations of revenue also enter in.

Consider the short-period equilibrium of the competitive industry (Bk V, Ch. V:4). The representative firm is a *price-taker* but *not* in the sense of being an 'atomistic competitor' who knows

the market price *prior* to deciding how much to produce. It finds out what price its catch will fetch only when the catch of the whole fleet has been landed. Since the boat owners filter out day-to-day shocks, a non-linear oscillatory process is not likely and smooth convergence to short-period equilibrium may be presumed. Obviously, this process does not require perfect knowledge, much less perfect foresight. Neither does it require large numbers of firms with identical cost-curves producing perfect substitutes. The rivalrous process might well have boat-owners jockeying for advantage by going to different fishing grounds for a higher-paying mix of fish and so on. Again, it is worth noting that experimental results have shown that large numbers of competitors are not necessary to obtain the competitive result.

George Stigler in his famous paper on the development of the theory of perfect competition expressed some puzzlement over the fact that Marshall had not really contributed to it. However, while Marshall made considerable use of the concept of a *perfect market*, that is, a market where all participants deal at the same price, later concepts of *perfect competition* are irrelevant to the equilibrium of his feedback-governed adaptive market process. In his competitive equilibrium, the (representative?) firm has its marginal and average costs equal to the price *received*. The condition is stated in *ex post* terms. The firm *expects* its costs and revenue to be remain what it is and the just realized outcome confirms this expectation. So it stays put. But it does not see itself as facing a "horizontal demand schedule."

It is one of the virtues of Marshall's approach that since it does not trap us into perfect competition neither does it force us to escape into imperfect competition.

Towards the Long Run

If on the average the firms in the industry earn above normal profits, the industry will add to capacity. If this process is allowed to run its course without further shocks to demand (or technology), it would reach a long-period equilibrium where quasi-rents have been eliminated.

This, it might seem, just adds another differential equation -- another 'law of motion -- to the dynamics of the Marshallian system. But the dynamics underlying this extension of his *static method* are not as tidy as those of his short-period analysis.

Expanding capacity is *not* a routine adaptation to a fairly well-known environment. Even assuming no radical changes in technology, growth must force firms to learn to cope with a changing

environment. It is not just a matter of a proportional scaling up of inputs and thus output. Economies of scale are ever present. The firms in the expanding industry will in varying degrees realize a mixture of internal and external economies that none of them foresees accurately at the outset. In his long-period analysis, Marshall strived hard, as Shackle (1972, p. 287) put it, to "spin together ... the mutually repellent strands of rationality and novelty."

Investment does not simply take the form of adding plant and equipment: "A great part of the capital invested in a business is spent on building up its internal organization and its external trade connections" (Bk. V, v, 7). This is the part of the capital that reflects the life-cycle of the firm -- growing when it prospers, shrinking when it declines, and disappearing altogether when it fails.

The notion of a representative firm also undergoes a rather subtle change as we take it from the short to the long period. We might imagine a short-period representative firm, such as was proposed above, that would actually exist. It is typical not only in being at rest when the industry is in equilibrium but also (presumably) in being "average" in some respects (size, variable cost, etc). Similarly, the firm that is representative of the long-run industry equilibrium should be at "high noon" of its life cycle where the "forces of progress and decay" are balanced. But it is not an average or typical firm in any other respect:

"We cannot then regard the conditions of supply by an individual firm as typical of those which govern the general supply in a market (p. 380) .... [because] the causes which govern the facilities for production ... of a single firm ... conform to quite different laws from those which control the output of an industry" (p. 379).

In the long period context, the representative firm is thus a purely mental construct, an aid to organizing the analysis, but without empirical counterpart.

Keynes and Marshall

John Maynard Keynes had been a pupil of Alfred Marshall and his price theory was in all essentials Marshallian. Prior to Keynes, it was however generally taken for granted that as long as Marshall's "laws of motion" were universally obeyed (that is, in the absence of "rigidities" or "inflexibilities" of any kind), the economy as a whole must surely gravitate to a full employment equilibrium. Keynes came to realize that this presumption was no more than a presumption and his *General Theory* was in effect a "revolt" against this central tenet of Marshallian neoclassicism

(Leijonhufvud, 1999).

Marshall for Our Time?

Marshall's adaptive and evolutionary theory could not be formalized with the mathematical tools at his disposal. His "static method" is an ingenious attempt to model central elements of it. He pressed the attempt as far as it would go -- he himself was frequently apprehensive that he had pressed it too far: "...it is especially needful to remember that economic problems are imperfectly presented when they are treated as problems of statical equilibrium and not of organic growth"(p. 382). For many decades, his theoretical conception could not be developed beyond the point to which he had taken it.

In more recent times, complex systems theory has progressed far beyond the state of the arts in Marshall's time. Moreover, computer modeling makes it possible to study the behavior of complex dynamical systems for which it is not possible to find analytical solutions. There are good reasons to revisit Marshall's theory *while abandoning the crutches of his static method* for there is much in the architecture of his system that could provide design for *agent-based* models. General equilibrium theorists have been rather scornful of Marshall's partial equilibria, but in a future economics freed from the equilibrium straightjacket and devoted to the analysis of processes, the modular architecture of Marshall is right and direct general interdependence wrong. The Marshallian tradition may yet make a comeback.

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