February 2014

Greenhouse Gas and Cyclical Growth

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Abstract A growth model incorporating dynamics of capital per capita, atmospheric CO_2 concentration, and labor and energy productivity is described. In the "medium run" output and employment are determined by effective demand in contrast to most models of climate change. In a "long run" of several centuries the model converges to a stationary state with zero net emissions of CO_2. Properties of dismal and non-dismal stationary states are explored, with a latter requiring a relatively high level of investment in mitigation of emissions. Without such investment under "business as usual" output dynamics are strongly cyclical in numerical simulations. There is strong output growth for about eight decades, then a climate crisis, and output crash.

JEL codes:

Key words: Demand-driven growth, climate change, simulation

JEL classifications: E12, E2, Q54

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* New School for Social Research. Research supported by the Institute for New Economic Thinking (INET) under a grant to the Schwartz Center for Economic Policy Analysis at the New School. Thanks to Nelson Barbosa, Jonathan Cogliano, Gregor Semieniuk, Rishabh Kumar, Codrina Rada, and Armon Rezai for invaluable contributions.

How will accumulation of greenhouse gas (GHG) and economic growth affect each other over many decades? In what ways will output, income distribution, and employment respond to rising levels of atmospheric CO_2 concentration? In this paper we address these questions, central to our times, by breaking from the mainstream consensus and assuming that economic activity is determined by aggregate demand in the "medium" run. The "long" run is set up as a steady state in which demand and supply effects commingle.

Standard models address the climate question strictly from the supply side, nearly always in a Ramsey optimal savings framework. This formulation raises several problems.

Given the havoc that climate scientists expect from global warming, the full employment assumptions built into supply side models strain credibility.

As will be seen, even with assumed full employment a Solow-Swan growth model linked with GHG accumulation generates complicated cyclical dynamics. Optimal growth models suppress cyclicality and use investment in "mitigation" of emissions to generate smooth trajectories of capital per capita and atmospheric GHG concentration toward a steady state. Using optimization to build such smooth behavior into a model's solutions is not necessarily wise. It elides dynamical complications and does not clarify how mitigation may fit into practical policy decisions.

We have argued elsewhere (Foley et al., 2013) that the smooth paths of state variables in optimizing models tend to be accompanied by strongly fluctuating values of the costates, i.e. the asset prices associated with capital and GHG concentration. Consequently, implicit interest rates vary strongly over time, with dynamics dependent on the detailed specification of a model. Because climate change is not a “small” perturbation to the economic system, the optimizing approach fails in its primary task of calculating a *constant* “appropriate” discount rate and social cost of carbon.

The key components of the demand-driven model are accumulation equations for atmospheric concentration of GHG (treated herein as CO_2 only) and capital per capita. Labor productivity also enters the specification as a third dynamic variable.

There is no aggregate production function with associated marginal conditions, so medium run output has to be determined by effective demand. A specific formulation relating demand to the functional income distribution is provided. Two channels are considered for the effects of GHG concentration on the real economy – either a reduction in profits and therefore investment demand or capital stock destruction as induced by faster depreciation. “Damage functions” for the transmission of these effects are described informally in the text with details in an appendix.

In the central dynamic equations, higher capital per capita increases output which in turn increases the speed of CO_2 accumulation. On the other hand, higher atmospheric GHG concentration reduces output and growth of capital per capita. Hence we have a variation on “typical” predator-prey dynamics – CO_2 is the predator and capital per capita the prey. Numerical simulations suggest that there may be an upswing in capital per capita for around eight decades, followed by a crash *of output and capital only.* Contrary to familiar fox-and-rabbit models, the decay rate of CO_2 in the atmosphere is *very* slow (the “fox” is almost immortal). *Concentration remains high*, blocking any chance of economic recovery.

We follow the usual growth theory convention of setting up a model that converges to a steady state. In practice, the system *must* converge to a *stationary* state with constant capital stock, CO_2 concentration, productivity, etc. Otherwise, CO_2 accumulation would overwhelm the system. As in the optimal growth models investment in mitigation can offset the crash, and lead to a non-dismal stationary state. In numerical simulations the share of output required is more than half total world defense spending and roughly double current worldwide energy consumption subsidies.

In the rest of this paper, we describe medium-term adjustment and sketch the accumulation equations as well as dynamics of energy and labor productivity. Steady state behavior of the model is then considered, followed by discussion of transient dynamics toward the steady state in “business as usual” (BAU) and “mitigated” scenarios. Implications for employment and distribution are pointed out.

Macroeconomic relationships

There are three dynamic variables: atmospheric CO_2 concentration in parts per million by volume or ppmv (G), capital stock per capita (κ), and the output/labor ratio (“labor productivity” ξ). The increase in G (or $\dot{G}$) is proportional to output (X) with the factor of proportionality reduced by outlays on mitigation (m) as a share of X. The increase in κ (or $\dot{\kappa}$) is driven by the investment/capital ratio ($g = I/K$) less depreciation (rate δ) and the population growth rate (n). In line with the literature on ecological economics the labor productivity growth rate ($\hat{\xi}$) depends on the growth rate of “energy intensity” or the energy/labor ratio ($e = E/L$).

As noted above, the long run must necessarily take the form of a stationary state with $\dot{G} = \dot{\kappa} = \hat{\xi} = \hat{e} = n = 0$. That is, G, κ, X, capital stock (K), employment (L) and population (N) must all be constant.

In the medium run X and L are determined by effective demand driven by g and m. "Capital utilization" $u = X/K$ *increases* with the profit share π via an increase in investment demand $g(\pi)$ so $u = u(\pi)$. In the usage of contemporary Keynesian growth theory (Taylor, 2004) demand for output is "profit-led." On the other hand, π is assumed to fall in response to tighter labor market as signaled by in an increase in the labor/population ratio $\lambda = L/N$.[1] Macro stability follows automatically from this profit-led/profit-squeeze combination.[2]

A key identity $\lambda = \kappa u/\xi$ ties the medium run together. Along with the investment function and macroeconomic balance it implies that higher κ increases λ, and reduces π and g. Lower g means that growth of κ slows, i.e. $d\dot{\kappa}/d\kappa < 0$ so growth in capital per capita is (locally) dynamically stable. Capital stock scales the system. In contrast to neoclassical supply-side models there is no aggregate production or cost function although the identity $\lambda = \kappa u/\xi$ and the assumption $\partial\pi/\partial\lambda < 0$ do apply.

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[1] Here we follow Marx who in several passages in *Capital* sketched a theory of business cycles (formalized a century later by Goodwin, 1967) pivoting on shifts in the functional income distribution. At the bottom of a cycle, the real wage is held down by a large reserve army of un- or under-employed workers, and capitalists can accumulate freely. However, as output expands the reserve army is depleted and λ goes up. The real wage rises in response to a tighter labor market, forcing a profit squeeze. To keep the analysis to low dimensionality we omit detailed discussion of the dynamics of such a cycle.

[2] In a Solow-Swan or Ramsey model incorporating full employment, medium-term adjustment takes the form of shifts in investment in response to changes in saving. Higher GHG concentration cuts directly into output by shifting the aggregate production function downward.

Two variants of this specification are considered. In one, π is also squeezed by G as rising CO_2 concentration cuts into profits. The linkages could include direct losses of output such as crops, higher non-wage costs of production, and weather-induced shortening of supply chains which magnify mark-ups (Kemp-Benedict, 2014). The appendix gives details.

In the other variant, π decreases just with λ but higher GHG concentration raises the depreciation rate ("capital destruction") and slows growth of κ. Lower capital stock reduces the level of output directly.[3]

Accumulation

Following the well-known "Kaya identity" from climate science (Waggoner and Ausubel, 2002), the accumulation equation for CO_2 is

$$\dot{G} = \chi E - \mu(m)X - \omega G = [(\chi/\varepsilon) - \mu(m)]X - \omega G \tag{1}$$

with $\varepsilon = X/E$ as energy productivity. Subject to decreasing effectiveness through the function $\mu(m)$ higher mitigation reduces the factor of proportionality of $\dot{G}$ with respect to X. The parameter ω is small – atmospheric CO_2 dissipates very slowly.

There is a steady state at

$$G = \omega^{-1}[\chi(e/\xi) - \mu(m)]uN\kappa = \omega^{-1}[(\chi(e/\xi) - \mu(m)]X \tag{2}$$

Note that steady state G is proportional to u, κ, and N (a Malthusian touch) or steady state X.

The other key accumulation equation is for capital per capita

$$\dot{\kappa} = \kappa(g - \delta - n)\ . \tag{3}$$

[3] As noted above, capital stock K scales the system. In our simulations, when the negative effect of κ on π is taken into account the elasticity of X with respect to K is 0.75. The medium-run multiplier is 1.7.

There is a steady state when

$$g = \delta + n \tag{4}$$

and a stationary state when $n = 0$.

An ostinato theme in ecological economics is that labor productivity is closely tied to energy intensity – Figure 1 provides a recent illustration relating the growth rate of ξ to the growth rate of e. Hence we assume that producers choose a growth path for e that converges to steady state level, and labor productivity growth is determined as

$$\dot{\xi} = \xi T \hat{e} \tag{5}$$

with $T = 1.5$. Energy productivity for use in (1) is set by the identity $\varepsilon = \xi / e$. A value of $T > 1$ assures that ε and e increase together, in line with much recent data.

Figure 1

Steady states

For numerical illustration we assume constant steady state levels of population (initial level = 7 billion, final = 10), energy intensity $e = E/L$ (initial = 4 kilowatts per employed worker, final = 6) and labor productivity $\xi = X/L$ (initial = $20,000 per unit of labor, final = $35,000).

In the first medium term variant, the steady state condition (4) with $n = 0$ means that $g = \delta$ determines π from investment demand. Then π sets u from macro balance. As described above we have $\pi = F(\lambda, G) = F(\kappa u / \xi, G)$ with negative partials from the profit squeezes. Hence in steady state G and κ must trade off along a “nullcline” in the (κ, G) plane to hold π constant.

As shown in Figure 2, the slope of the nullcline for $\dot{G} = 0$ is sensitive to m so that mitigation can support a non-dismal steady state. With no mitigation, $\kappa < 20$ and $G = 759$ in a dismal BAU steady state. (Initial values are $\kappa = 28.57$ and $G = 400.$) The mitigated steady state $G = 486$ might correspond to $2.5°\mathrm{C}$ of global warming over the pre-industrial baseline – more than the currently accepted "red line" of 2.0˚. A steady state with $G = 759$ would mean 5 or $6°\mathrm{C}$ of warming,[4] a recipe for disaster.

Figure 2

If time trends for population and productivity were ignored, phase diagram aficianados would infer that from an initial position of $\kappa = 28.6$ and $G = 400$ the BAU solution trajectory would follow a counter-clockwise path toward the steady state. Both variables would rise until they hit the $\dot{\kappa} = 0$ nullcline. Thereafter κ would start to fall and G to rise until they hit the $\dot{G} = 0$ nullcline and then both would spiral toward the steady state. In a mitigated solution, κ would rise with G initially falling slightly and then rising in a converging spiral. With time trends included both patterns are illustrated in simulations below.

Table 1 gives values for the Jacobian matrices for κ and G at the BAU and mitigated steady states. The values along the main diagonals show that convergence will be slow for κ and (especially) G. In the BAU solution the cross effects $\partial\dot{G}/\partial\kappa > 0$ and $\partial\dot{\kappa}/\partial G < 0$ are relatively strong; the magnitudes drop off in the mitigated solution. These results feed into the discussion below of dynamics away from the steady states.

[4] The rule of thumb is that after an atmospheric lag of about 10 years doubling pre-industrial CO_2 concentration of 280 ppmv should lead to a temperature increase of about 3˚C. As of 2013 the increase is already about 0.8˚ and getting to 1.3˚ is well underway. The numbers in the text follow accordingly.

Table 1

In the second medium run variant (Figure 3), there is no direct adverse effect of G on π, but higher CO_2 concentration raises the depreciation rate δ – there is more rapid destruction of capital stock. Now in steady state, higher G and δ must lead to higher π (investment is profit-led). But a higher π must be associated with a lower κ via lower λ. Again we get a trade-off between G and κ. Mitigation can still support a high level steady state. No mitigation leads to low level stagnation. Similar results show up in, say, a Solow-Swan model in which $\delta = g = sf(\kappa, G)$ so δ determines κ from *supply side*. Damages from GHG accumulation follow from an assumption that $\partial f / \partial \mathrm{G} < 0$.

Figure 3

Table 2 gives a quick summary of steady state results for both variants. One can further show that higher steady state population strongly reduces κ and per capita output X/N under BAU; there is a relatively weak impact on G. The magnitudes reverse in mitigated solutions. Higher labor productivity (which also raises energy productivity) increases κ, G, and X/N, more strongly in mitigated solutions

Table 2

Transient paths to steady states

We set up simulations to track dynamics of κ, G, and ξ toward steady states. Growth trajectories are affected by assumed rates of increase of population, labor productivity, and energy intensity (modeled as logistic curves between initial and final levels). We first look at BAU growth when there is an adverse effect of CO_2 concentration on profitability (with similar results when higher concentration increases the depreciation rate). Figure 4 shows trajectories of variables of interest over a five century time span.[5]

Figure 4

Along the lines discussed above, the model generates cyclical dynamics. Capital per capita and output rise for about eight decades, and then crash. Apart from energy and labor productivity levels which rise according to (5) the other economic variables follow a similar pattern. Output becomes constant near its initial level of $60 trillion so output per capita falls by around 35% at a final population level of 10 billion.

Atmospheric CO_2 concentration stabilizes at well over 700 ppmv, leading to a big temperature increase as noted above. There is no possibility for output per capita to recover. Two potential offsets are weak.

First, there could be a reduction in the emissions/output ratio χ in (1) due to a shift in the mix of fossil fuels in use away from coal and oil toward natural gas. This change allows a modest reduction in CO_2 accumulation which permits better economic performance but the basic BAU oscillation persists.

The cycle also remains when there is slower growth of energy intensity, meaning that the growth rate of energy productivity will decrease as well. Then GHG concentration should grow faster because the ratio χ/ε will fall less rapidly in (1). On the other hand, labor productivity growth will also drop, cutting into capital accumulation and

[5] The model's differential equations were simulated using *Mathematica 9.*

output. The latter effect dominates. Economic performance deteriorates, marginally slowing the rise in greenhouse gas.

Next we turn to growth with mitigation at initial cost of $160 per metric ton of carbon, or $44 per ton of CO_2 (in the mid-range of current estimates).[6] Outcomes with mitigation outlays of 1.0% and 1.25% of world output ($60 trillion initially) are illustrated in Figure 5. With 1.25% mitigation, CO_2 concentration can be stabilized. This outlay is around one-half of current level of defense spending and roughly twice the level of worldwide energy consumption subsidies.

Figure 5

As illustrated in Figure 6, the macro economy with mitigation follows a growth path of capital per capita to a stationary state that lies a bit below the one that the model generates in the absence of global warming. The BAU and 1.25% mitigation scenarios broadly correspond to the highest and lowest damage paths in the IPCC (2007).[7] One can show that "front-loading" mitigation leads to more favorable results (G converges to about 400). A "climate policy ramp" with low initial mitigation levels that gradually rise would be harmful.

Figure 6

[6] "Mitigation" comprises many different activities – reducing motor vehicle use, increasing energy efficiency of buildings, carbon sequestration, conservation tillage, ending deforestation, etc. As with reducing the parameter χ or the growth rate of energy intensity (see above) the effect of any single step toward mitigation will be "small," but in total they can have a big impact. See Pacala and Socolow (2004).

[7] At the time of writing, the fifth IPCC report on mitigation (IPCC forthcoming) is not yet published.

The results in the demand-driven model are largely determined by convergence dynamics of κ and G to steady state levels. The same basic pattern appears under variant medium-run adjustments, e.g. higher CO_2 concentration reduces profitability *or* leads to capital destruction via faster depreciation *or* shifts down a neoclassical aggregate production function in a supply-driven full employment Solow-Swan growth model.

Figure 6 illustrates dynamics in a Solow-Swan scenario. The unconstrained solution for capital per capita lies below the demand-driven path, but under BAU both specifications converge to a similarly dismal stationary state. The full employment assumption partially stabilizes the dynamics but the cyclical rise and fall of κ going into the steady state persists. Finally, 1.25% mitigation returns the simulation close to its unconstrained variant.

Impacts on labor

The demand-driven model can be used to explore implications of global warming for labor.

An initial observation is that at the macro level, the real wage is equal to the labor share of output (or one minus the profit share π) multiplied by labor productivity ξ. Approaching a "long run" stationary state, π will stabilize so that the real wage will increase along with productivity growth.[8] This standard result from growth accounting suggests that under BAU the economy will tend toward a high wage, low employment equilibrium. Indeed the BAU steady state value of the employment/population ratio λ is

[8] This sort of convergence only shows up over centuries. Over decades the real wage can be stagnant or fall, as in the USA since the 1980s.

65% below its initial value due to high G, stagnating X and increases over time in labor productivity and population. In the mitigated solution, λ rises.

As in all demand-driven models away from steady state, employment is determined as a "lump of labor," or $L = X/\xi$. Alternatively, since $\pi = F(\lambda, G) = F(\kappa u/\xi, G)$, $L = \lambda N$ is a function of π, G, and N. The elasticity of λ with respect to ξ is -1 (higher productivity destroys jobs in proportion) in both BAU and mitigated steady states. It is about -0.8 along transient paths because X rises with higher ξ.

Bottom line

The intrinsic growth rate of capital per capita κ is low; the rate for atmospheric CO_2 concentration G is lower still. On the other hand, a higher level of κ (and output, employment, etc.) strongly stimulates growth of G. Ultimately higher G slows the growth of κ and will make it turn negative. Moreover, because natural dissipation is very slow, once G reaches a high level there is no way for κ to recover. Under "business as usual" in our model's simulations there will be a climate crisis followed by economic stagnation. The time scale of the dynamics is such that the crisis could occur within a now young person's lifetime. These results carry through under differing specifications of medium-term macroeconomic adjustment and capital formation.

The conundrum for policy is that for several decades – half a human life span – the impending crisis could be masked by ongoing growth (see Figure 4). Effective mitigation of CO_2 emission could prevent the crisis and support a fairly high world level of economic activity with zero net new emissions and stable income per capita (assuming that population growth slows to zero or less). The sooner a mitigation effort gets underway at a level exceeding one percent of world GDP, the better. If serious mitigation is not implemented soon, prospects for the world economy are dismal.

Appendix: functional forms and parameterization

Medium-term macroeconomic balance is determined by the equations

$$s(\pi) = s_w + (s_\pi - s_w)\pi \quad , \tag{6}$$

$$g(\pi) = g_0 + \alpha\pi u \quad , \tag{7}$$

and

$$g(\pi) + h + mu - s(\pi)u - \tau u = 0 \tag{8}$$

with h as the ratio of government spending to capital and τ as a tax rate.[9] In steady state (6)-(8) can be solved for π and u, given g; in the medium run for g and u, given π.

In the first medium-term adjustment scenario, we adopted the functional form

$$\pi(\lambda, G) = [\Phi Z(G)]^B \lambda^{-A} \tag{9}$$

with

$$Z(G) = [1 - (\tfrac{G-280}{\bar{G}-280})^{1/\eta}]^\eta \quad . \tag{10}$$

Here, $Z(G)$ is a concave decreasing damage function carried over from a supply-driven climate model constructed by Rezai et al. (2012) with 280 ppmv as pre-industrial CO_2 concentration and $\bar{G} = 780$ as a level at which extremely severe climate damage occurs. Figure 7 illustrates the damage function for various values of the parameter η. After experimentation with sensitivities, in simulations we set $\eta = 0.5$.

Figure 7

The second scenario uses the equation

[9] One has to carry government spending and taxes in the accounting to fit the data and generate plausible multiplier values.

$$\delta = 0.016 + 0.00009G$$

which sets $\delta = 0.052$ at $G = 400$ and $\delta = 0.07$ at $G = 600$. The effect of labor market tightness (and capital per capita) on the profit share is given by the S-shaped relationship

$$\pi(\lambda) = 0.2 + 0.4/[1 + e^{\psi(\lambda-\lambda_0)}] \quad (11)$$

which implies that π decreases from 0.6 to 0.2 as λ increases.

The functional form for decreasing effectiveness of mitigation in (1) is

$$\mu(m) = \psi\frac{1-e^{-\phi m}}{\phi}. \quad (12)$$

For equations (6)-(8), world output or GDP is set at roughly $X = 60$ (trillion dollars). With $K = 200$, $u = 0.3$. With the share of government spending in output $H/X = 0.33$, $h = 0.099$. If the fiscal deficit is normally three percent of GDP, then $\tau = 0.3$. A plausible level of the world saving rate is $s(\pi) = 0.24$. If $m = 0$ initially, then the investment/capital ratio becomes $g = 0.063$. The profit share of output is roughly $\pi = 0.4$. If the saving rate from profits is $s_\pi = 0.28$ then the rate from wage income becomes $s_w = 0.2133$. With $\alpha = 0.25$, $g_0 = 0.033$ supports the investment function. Aggregate demand will be profit-led if $\alpha + s_w - s_\pi > 0$, a condition satisfied by these numbers.

For use in (9)-(10), we set world employment $L = 3$ (billion) and population $N = 7$ so that $\lambda = 0.42857$. With current output of 60, $\xi = 20$. The capital/population ratio is $\kappa = 200/7 = 28.5714$ (or \$28,571 per capita). Together with $\eta = 0.5$ we set $A = 2$ and $B = 2$. The parameter Φ was “calibrated” to fit $\pi = 0.4$, taking the value of 0.2792.

In (11) we set $\psi = 18.66$, broadly consistent with observed ranges of the profit share.

In (1), it is simplest to think of energy use in terms of terawatts of power (as opposed to exajoules of energy per year). The current world level is about 15 terawatts, of which 12 are provided by fossil fuels. Fossil fuel energy productivity becomes $\varepsilon = 60/12 = 5$. This energy use generates about 7 gigatons of carbon emissions per year, corresponding to an increase in G of 3.37 ppmv. The observed increase is about 2 ppmv, so that $\hat{G} = 2/400 = 0.005$ with atmospheric dissipation of 1.37 ppmv. The dissipation coefficient becomes $\omega = 1.37/400 = 0.0034$.

Assuming that there is now no effective mitigation or $\mu(m) = 0$ the growth rate of G becomes

$$\hat{G} = (\chi/\varepsilon)\,\Gamma - \omega \quad .$$

With emissions of 3.37 and fossil fuel energy use of 12, the ratio $\chi = 3.37/12 = 0.2808$ and $\nu = \chi/\varepsilon = 0.0562$. The balance equation for $\hat{G}$ works out to be

$$0.005 \approx (0.0562)(0.15) - 0.0034 = 0.0084 - 0.0034 \quad .$$

At present, G increases at a slower rate than κ.

In (12) if one percent of output (or \$0.6 trillion) is devoted to mitigation ($m = 0.01$) we have

$$\frac{1-e^{-0.06}}{6} = 0.0097 \quad .$$

That is, the cost-effective outlay is $(0.6)(0.97) = 0.582$.

A fairly high estimate of the cost of removing one ton of carbon emissions is \$160, or \$44 per ton of CO_2 (roughly twice the level now being considered by the government in the USA, according to Ackerman and Stanton, 2012). To reduce atmospheric CO_2 concentration by 1.0 ppmv would require removal of 2.07 gigatons of carbon from emissions at a total cost of $(2.07)(\$0.16\ \text{trillion}) = \$0.331$ trillion. Spending one percent of output would mitigate $0.582/0.331 = 1.7583$ ppmv. So we get the change in emissions as $-\Delta\dot{G} = 1.7583 = 0.582\psi$ or $\psi = 3.0211$.

In the Solow-Swan variant there is a neoclassical aggregate production function (Cobb-Douglas for simplicity)

$$X = Z(G)A(\xi\lambda N)^{0.6}K^{0.4}$$

with A as a calibration parameter. The employment/population ratio λ is no longer an adjusting variable but stays fixed at its base year level (a full employment assumption) and the damage function $Z(G)$ is the same as in the demand-driven version. If $v = X/N$ this equation becomes

$$v = Z(G)A(\xi\lambda)^{0.6}\kappa^{0.4} \quad .$$

Replacing (3) the growth equation for κ is

$$\dot{\kappa} = (s + \tau - m)v - (h + n + \delta)\kappa$$

which can be solved using the base year values of λ and G along with the model's equations for $\dot{\xi}$ and $\dot{N}$.[10]

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[10] Textbook presentations set $\kappa = K/\xi L$ but we avoid this specification to maintain comparability with the demand-driven model.

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Table 1

Steady State Jacobians		
BAU		
	κ	G
$\dot{\kappa}$	-0.0570091	-0.0198071
$\dot{G}$	0.102817	-0.0163718
1.25 % mitigation		
	κ	G
$\dot{\kappa}$	-0.0418889	-0.00261627
$\dot{G}$	0.0217864	-0.00376279

Table 2

Initial and steady state values for BAU and mitigated paths for two model versions			
	Initial value	BAU	Mitigated
1) Profit share decreases with both κ and G			
G	400	759.4	486.2
κ	28.6	19.8	63.0
X/N	8.6	5.6	18.3
λ	0.429	0.153	0.5
2) Depreciation rate increases with G			
G	400	698.6	464.7
κ	28.6	20.3	57.3
X/N	8.6	6.6	17.2
λ	0.429	0.181	0.468

Figure 1:

Evolution of Average Energy Use per Labor Growth Rate $(\widehat{E/L})$ vs. Labor Productivity Growth Rate $(\widehat{X/L})$ from 1971-90 (red) to 1990-2011 (purple).

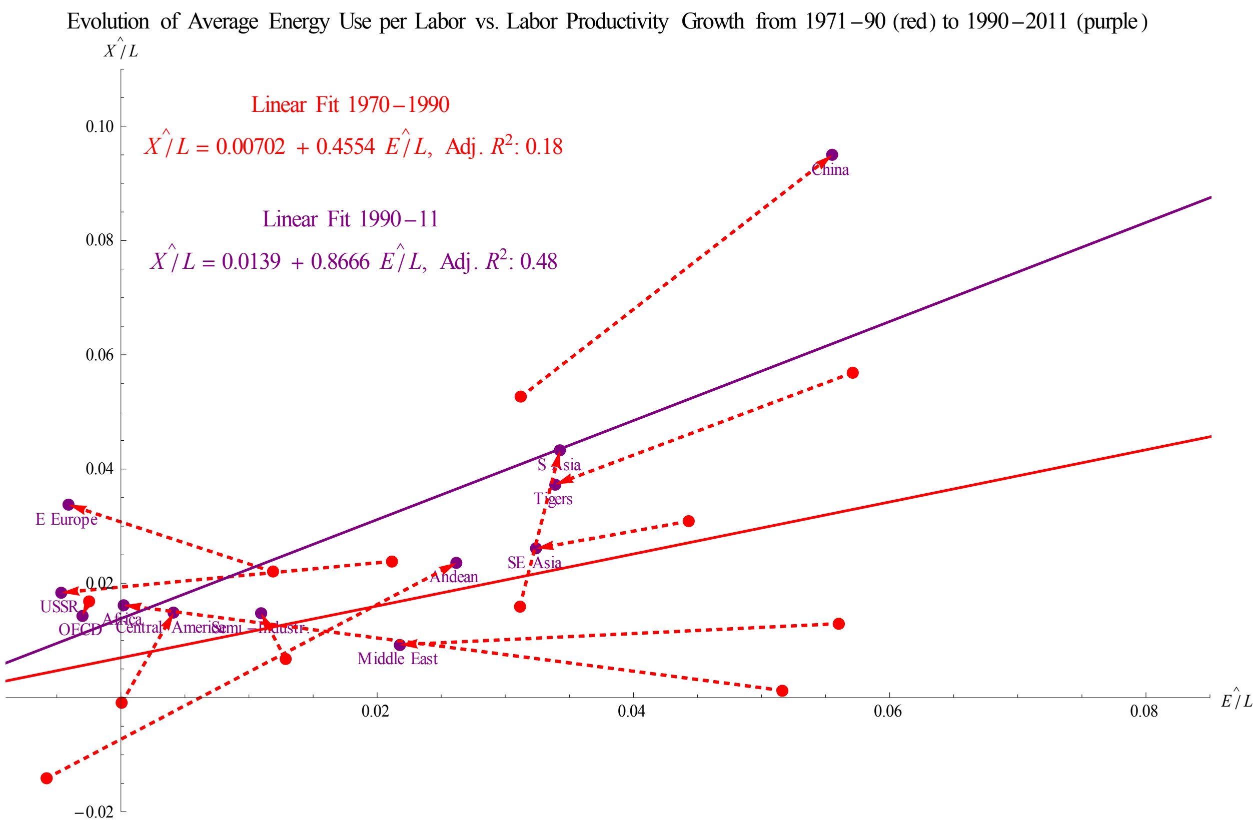

Figure 2:

Nullclines for per capita capital stock (κ) and CO_2 concentration (G) when the profit share decreases with both G and κ.

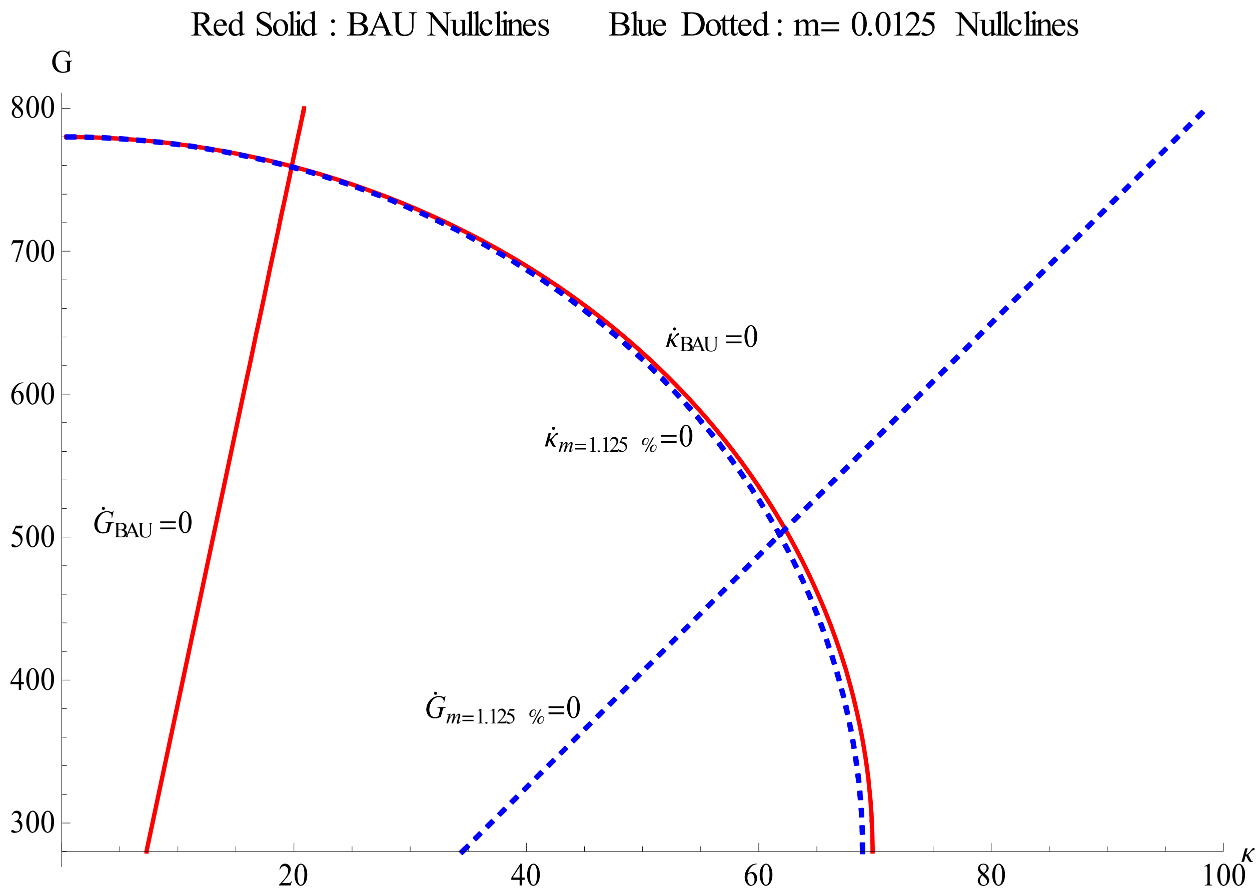

Figure 3:

Nullclines for capital stock per capita (κ) and CO_2 concentration (G) when higher G increases capital depreciation.

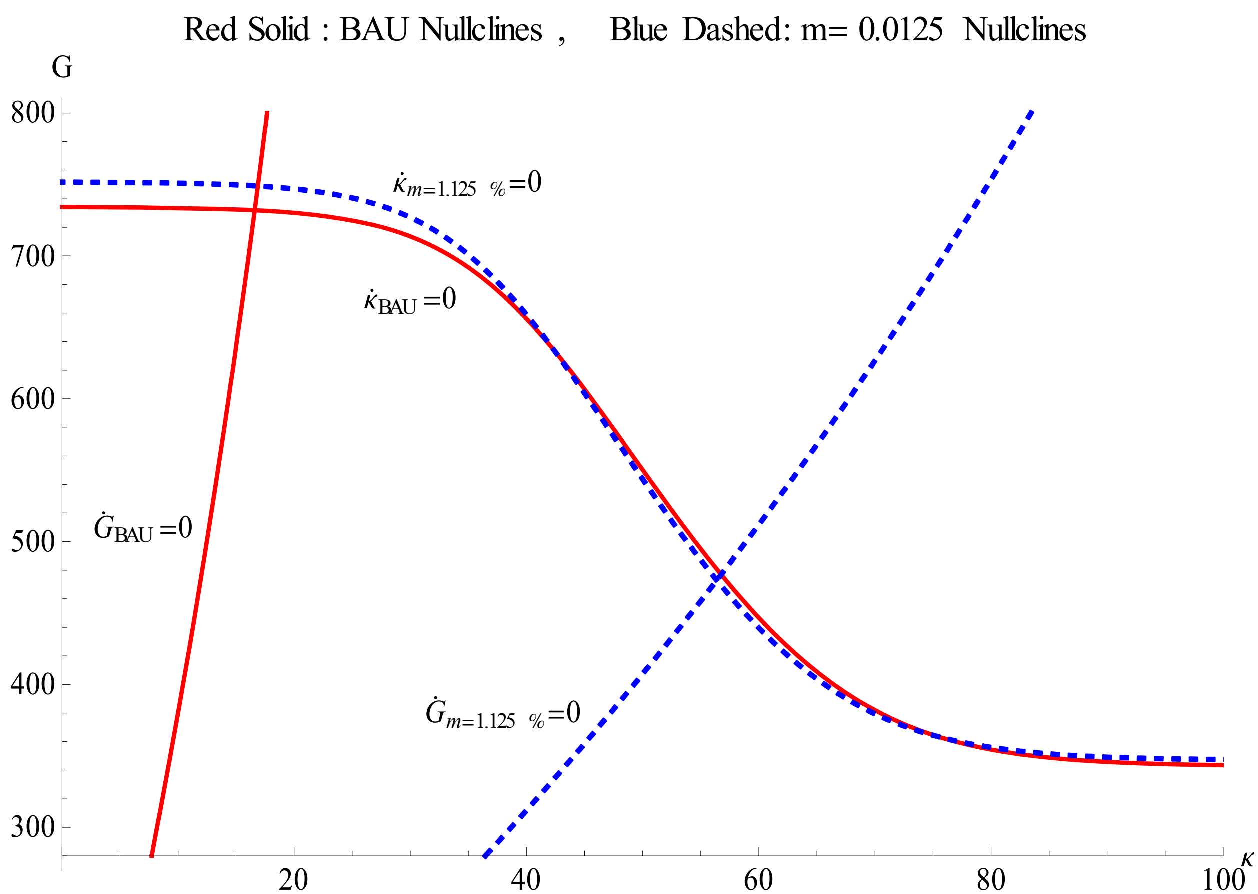

Figure 4:

BAU simulation when the profit share decreases with both κ and G.

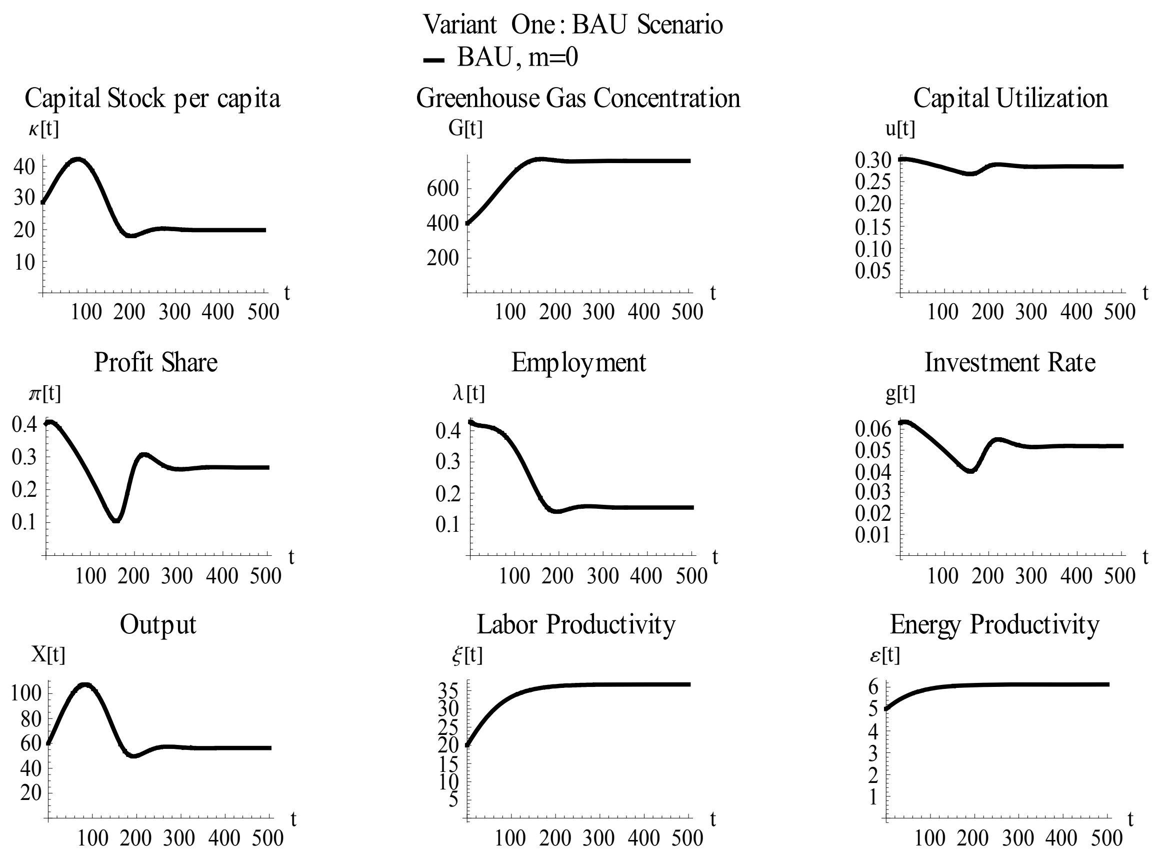

Figure 5:

BAU and mitigation simulations when the profit share decreases with both κ and G.

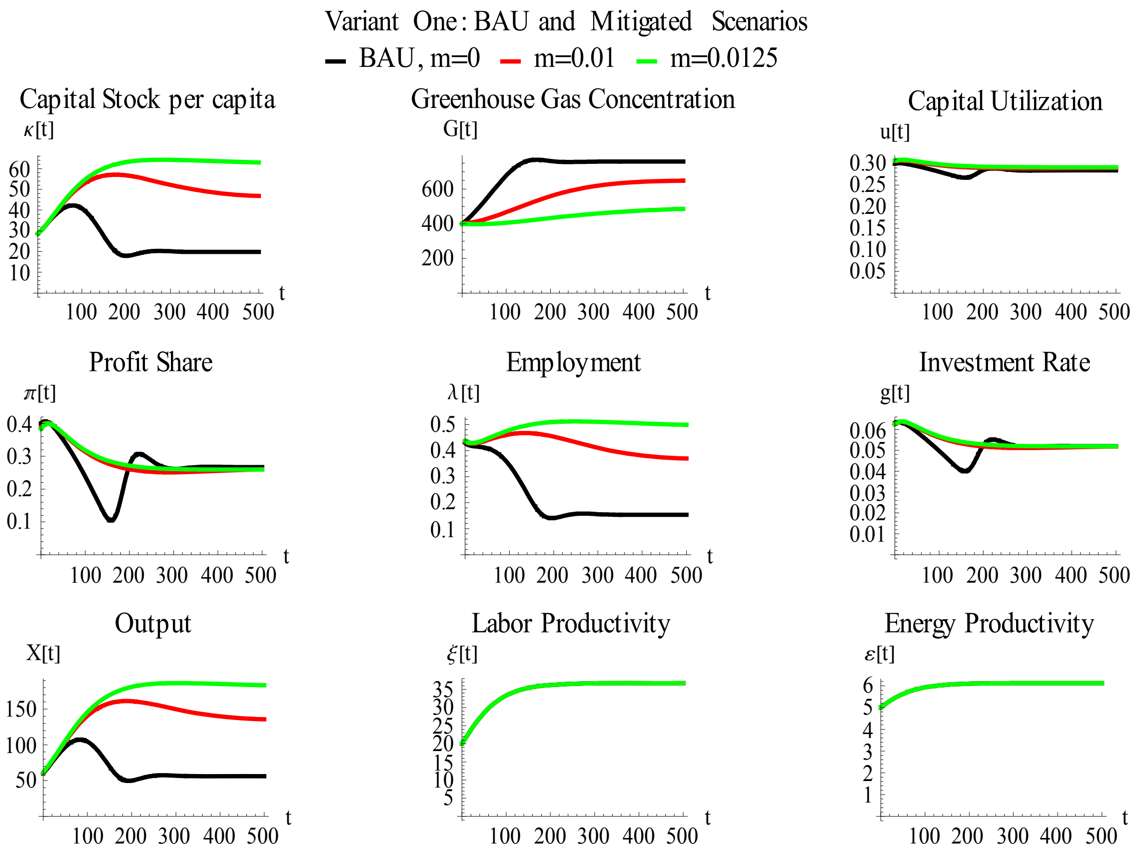

Figure 6:

Dynamics of κ for demand-driven and neoclassical growth spcifications.

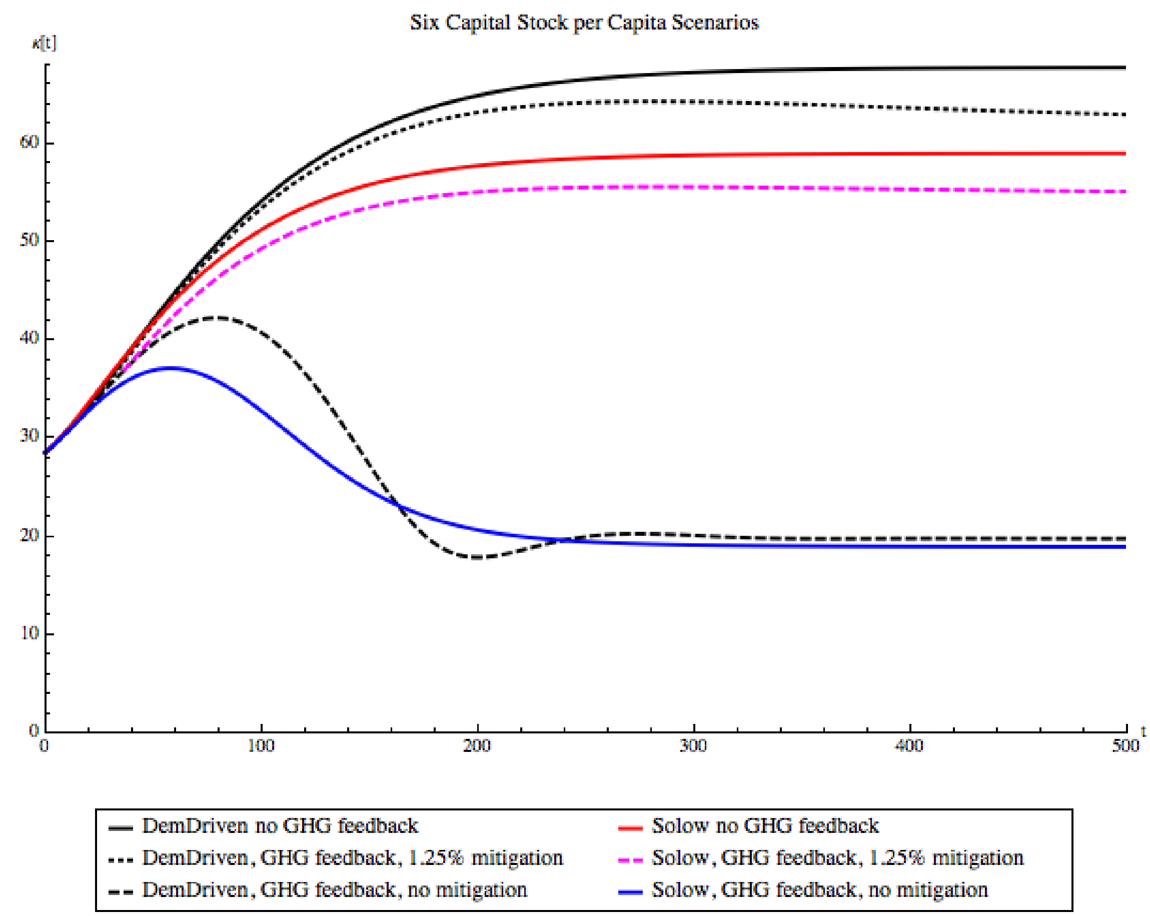

Figure 7:

Damage function for the profit share.