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A New Rational Expectations Hypothesis:

What Can Economists Really Know About the Future?*

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1 Overview

John Muth proposed the Rational Expectations Hypothesis (REH) to represent how the market (an aggregate of its participants) understands and forecasts outcomes. REH imposes internal consistency between the market's forecasts and "the relevant economic theory" (Muth 1961, p. 316). In implementing REH, economists have relied on a class of models that rest on a key premise: an economist can ignore the importance of quantitative changes in the economy's structure that he himself could not specify *ex ante* with a probabilistic rule. In the early 1970s, Robert Lucas pointed out that once such models are upheld as "the relevant economic theory," it follows on purely logical grounds that REH models represent rational forecasting and its implications for market outcomes.

There is ample evidence that quantitative structural change is an important driver of outcomes in real-world markets, most notably those for assets. The evidence also shows that this change often is triggered by historical events that are not exact repetitions of similar events in the past, for example, the appointment of a new Federal Reserve chair or company CEO. The timing of these events may be unknown, and their quantitative impact on the economy's structure depends on the extent of their novelty and the particular historical context in which they occur. As such, no one can fully anticipate – even in probabilistic terms – the structural change triggered by such events.

In our earlier work, we traced REH models' epistemological and empirical difficulties to their premise that "the relevant economic theory" can ignore the importance of unanticipated structural change in real-world markets.[1] The reason is simple: a rational individual understands the importance of such change. Profit-seeking compels him to be on guard for unanticipated structural change and revise his forecasting strategies accordingly. To rule out this behavior, as REH does, presumes that a rational individual foregoes profit opportunities. In this paper, we illustrate the irrationality of REH in real-world markets using a simple model of stock prices.[2]

Lucas (1976) and Sargent (1981) persuasively argued that explaining longer-term regularities in time-series data and analyzing the conse-

[1] See Frydman and Goldberg (2007, 2011, 2013a).

[2] In proposing internal consistency as a way to represent the market's forecast, Muth explicitly recognized that such representations should not be viewed as a normative hypothesis about how rational profit-seeking individuals should forecast the future. Indeed, he proposed REH as a "purely descriptive hypothesis." As he put it, "At the risk of *confusing* this purely descriptive hypothesis with a pronouncement as to what firms ought to do, we call such expectations 'rational"' (Muth, 316, emphasis added).

quences of alternative government policies requires that macroeconomics and finance theory be compatible with rational decision-making. This groundbreaking insight leads us to explore how theory can represent rational forecasting in real-world markets, where unanticipated structural change is an important factor driving outcomes. We call our approach a New Rational Expectations Hypothesis.

NREH retains the seminal idea that underpins REH: it imposes consistency within an economist's model. However, its "newness" consists an alternative core premise: an economist recognizes that the process underpinning outcomes will change at times and in ways that he cannot fully specify in advance, even with a probabilistic rule. This premise recognizes that there are immanent limits to what economists and market participants can know about the future.[3] Acknowledging these limits is crucial for ridding macroeconomics and finance theory of the irrationality that is inherent in the original formulation of REH.

We formalize NREH's core premise with models that are partly open to such unanticipated structural change. Partly open models serve as NREH's "relevant economic theory."

No one can specify *ex ante* the probabilistic rule and a particular set of quantitative structures that will characterize how outcomes will actually unfold over the indefinite future. A partly-open model nonetheless hypothesizes *ex ante* that the quantitative structures that will be needed to account for the process underpinning future outcomes share certain qualitative features. NREH formalizes these features as constraints on structural change that leave partly open when and how the relevant set of quantitative structures might change.[4]

As these constraints are unfamiliar, we develop them in the context of the well-known present-value model of stock prices. In order to highlight the key aspects of NREH and facilitate its comparison with REH, we set the discount factor to a constant and focus on representing the market's forecasts of dividends and stock prices in terms of fundamental factors, such as company earnings.

[3] Our emphasis on the inherent limits to knowledge about social phenomena is related to Soros's (1987, 2013) concept of fallibility. However, for Soros (2013, p. 311), fallibility stems from "[t]he complexity of the world... [which] exceeds our capacity to comprehend it... [and] the structure of the brain." Complexity makes it difficult to understand markets. But even simple systems, which our brains may easily understand at a point in time, undergo unanticipated structural change. This change is an important source of fallibility, and recognizing this is crucial for representing rational decision-making in mathematical models. For a comparison of Soros' conceptual framework with Imperfect Knowledge Economics, see Frydman and Goldberg (2013b).

[4] Our formulation of partly open models builds on Frydman and Goldberg's (2007, 2011) Imperfect Knowledge Economics.

To serve as "the relevant economic theory," a model must generate predictions for time-series data. It must therefore constrain unanticipated structural change in advance. Our partly open present-value model hypothesizes *ex ante* that there will be protracted intervals of time during which change in the structure of the dividend process will be moderate, in the sense that this change is constrained by a lower and upper bound. The model also hypothesizes *ex ante* that the intervals of moderate change will be punctuated by large structural changes that fall outside these bounds. The model does not constrain the timing of such large changes with a probabilistic rule and thus leaves open when intervals of moderate change begin and end.[5] The partly open constraints imposed by an NREH model are sufficient to generate implications for time-series data. They do so without representing future outcomes with an overarching probability distribution.[6]

The contrast with REH macroeconomics and finance models could not be sharper. Unlike NREH models, REH models "structure the world in terms of a 'market' that assesses probability distributions on future prices and then sets current prices on the basis of these assessed distributions" (Fama, 1976, p.168).

Fama acknowledges that these probabilistic representations of future prices are "approximations." He also recognizes that a probabilistic approximation is relevant for understanding and predicting future prices only if it "can be taken as true, at least until a better approximation comes along" (Fama 1976, p.142). Over the years, however, this pragmatic reason for relying on probability distributions to represent future prices *ex ante* has morphed into an unfounded premise that such approximations can indeed represent the "true" course of future outcomes.

This premise led macroeconomists and finance theorists to embrace the belief that they could derive exact probabilistic implications of rational decision-making that would be relevant in explaining both the past and the future. Researchers could then test these implications by estimating an REH model's probabilistic representation with *ex post* data.

A vast majority of REH models constrain the structure of their *ex ante* approximations to be the same at each point in time. Econometric studies, therefore, also constrain the structure of their *ex post* approxi-

[5] This contingency concerning the timing of large unanticipated structural changes plays a crucial role in NREH's ability to represent rational decision-making in real-world markets. See Frydman and Goldberg (2015).

[6] As we make clear in Frydman and Goldberg (2015), a partly open model does imply a family of probability distributions at every point in time. But partly open models do not specify exactly how these distributions will change over time. Thus, they do not represent future outcomes *ex ante* with an overarching probability distribution. See also section 2.3.2.

mations to be time-invariant. However, when researchers test for structural change in past data, they always find it. Time-invariant models, therefore, do not represent rational forecasting. It is not surprising that they have given rise to many major "puzzles," especially in financial markets.[7]

This empirical record has led researchers to estimate REH models that allow for structural change *ex ante*. Hamilton (1988, 1994) formulated a seminal class of such models. A model in this class hypothesizes a set of distinct quantitative structures that relate outcomes, such as stock prices, to fundamental factors. It also specifies a Markov rule that governs switches between these structures. Researchers typically find that Markov-switching models provide better approximations to past data than time-invariant models.

But these studies typically do not address the question of whether the quantitative change that they estimate on the basis of their Markov-switching model and *ex post* data could have been anticipated *ex ante*, or, equivalently, whether their estimated model would provide an adequate probabilistic approximation of future outcomes.

An important exception is Engel and Hamilton (1990), which estimates a Markov-switching model for the exchange rate. They examine whether the market understood *ex ante* the probabilistic approximation of structural change that they estimated with *ex post* data. They concluded that the answer was no. This is what one would expect if the process underpinning outcomes experienced unanticipated structural change during the study's sample period.

Kaminsky (1993) provides early evidence that structural changes estimated on the basis of *ex post* data could not have been fully foreseen by anyone, even in probabilistic terms.[8] She argues that the appointment of a new Fed chair triggers structural change of the transition probabilities of a Markov-switching model for the exchange rate. Frydman et al. (2015a) provide evidence that about 20% of the fundamental factors that moved daily stock prices between January 4, 1993, and December 31, 2009, involved such non-repetitive historical events, including wars, election outcomes, and new technologies.

The importance of such events makes particularly problematic the

[7] The empirical anomalies include the Meese and Rogoff (1983) exchange-rate disconnect puzzle, Bilson (1981) and Fama (1984) forward discount puzzle, and Rogoff (1996) purchasing power parity puzzle. For evidence that these anomalies stem from unanticipated structural change, see Goldberg and Frydman (1996) and Frydman and Goldberg (2007).

[8] For evidence that structural change in models of stock returns are related to historical events that are to some extent novel, see Pettenuzzo and Timmermann (2011) and Ang and Timmermann (2012).

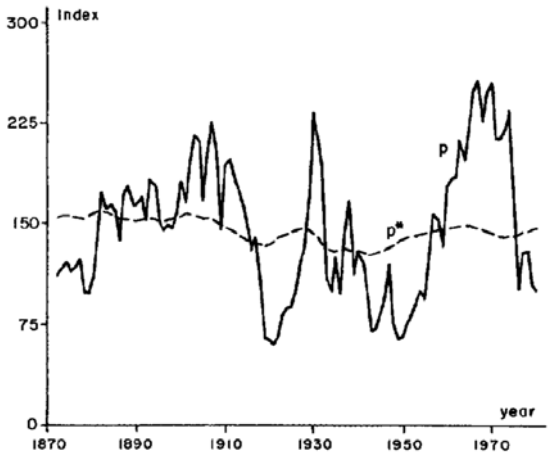

Figure 1: S&P500 Price Index (P) and its Fundamental Value (P*)

objective of using REH models to explain long-term regularities in time-series data and to analyze the consequences of alternative government policies. The longer the span of history from which the sample is drawn, the greater the likelihood that it will be marked by novel events. These events trigger structural change that, by design, an REH model's *ex ante* probabilistic approximation ignores. Like their time-invariant counterparts, any REH model that allows for structural change cannot represent rational forecasting.

Steadfast adherence to the premise that probabilistic approximations "can be taken as true" *ex ante* helps explain why decades of research by macroeconomists and finance theorists have left many empirical puzzles unresolved. Shiller's (1981) groundbreaking study provides perhaps the most striking early example of the difficulties that REH models have encountered in accounting for long-term regularities in time-series data.

As is well known, once any REH model's *ex ante* probabilistic approximation is "taken as true," *ex post* realizations of future prices should be the same as those predicted *ex ante* by the model (except for a random error term). Any REH present-value model predicts, therefore, that stock prices should be less variable than their fundamental value – the future discounted value of *ex post* dividends. However, Shiller found the exact opposite. His Figure 1, which we reproduce here, vividly shows that stock prices often undergo wide fluctuations around their fundamental value.

Behavioral-finance theorists have interpreted Shiller's and others' findings as evidence that participants' forecasts of dividends and stock

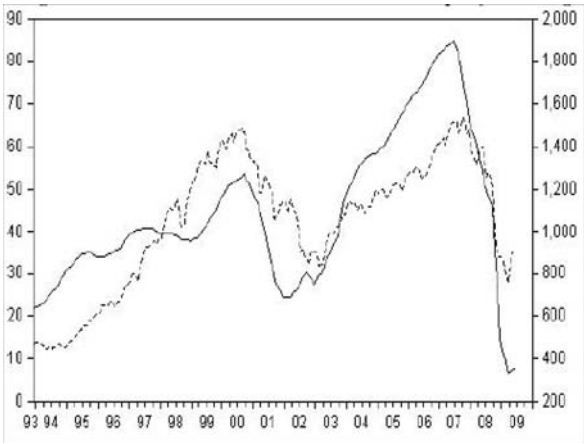

Figure 2: S&P500 Price Index (dotted line) and Company Earnings (solid line)

prices are driven by psychological factors or momentum trading that are largely unrelated to fundamental factors such as company earnings.

Our arguments here lead us to a very different explanation of the REH present-value model's empirical failure: the model fails not because participants in real-world markets are "irrational," as behavioral economists argue, but because they are rational. As Shiller and many others have found, unanticipated structural change renders REH's account of how stock prices are related to fundamental factors inconsistent with time-series data. Contrary to the behavioral view, however, even cursory observation suggests that fundamental factors are a major driver of stock prices.

Figure 2 reveals clearly that stock prices co-vary positively with earnings. The NREH present-value model generates such qualitative predictions. Consequently, we interpret Shiller's finding as evidence that REH versions of the model – by selecting one structure or a particular set of structures to represent outcomes *ex ante* – fail to account for how rational participants interpret news about fundamentals to forecast dividends and stock prices.[9]

NREH models' reliance on fundamental factors in explaining stock prices does not imply that psychological factors play no role. Indeed, opening the present-value model to unanticipated structural change implies that such factors are crucial for how market participants select a forecasting strategy and when and how they decide to revise it. Psychological factors play a role not because irrational participants rely on

[9]For empirical support of this interpretation, see Frydman and Goldberg (2015a).

them instead of fundamentals, as behavioral economists contend, but because they help rational participants interpret the impact of movements in fundamentals on stock prices.

Our NREH model's prediction that stock prices and earnings co-move positively over protracted time intervals does not necessarily imply that the model can explain why stock prices fluctuate widely around their fundamental value, as depicted in Figure 1. In this paper, we sketch how opening the present-value model to unanticipated structural change and representing rational forecasting with NREH could account for such fluctuations without presuming that market participants are irrational.[10] We leave a more complete NREH explanation of asset-price fluctuations for future research.

2 A Sketch of an NREH Present-Value Model

Macroeconomic and finance models relate market outcomes to one or more causal factors and characterize the process that governs those factors. This structure formalizes an economist's understanding of outcomes at a point in time. As time passes, the process that underpins both market outcomes and the causal factors may change, implying that distinct model structures may be required to represent how this process unfolds over time. In order for the model to generate predictions for time-series data, it must constrain structural change *ex ante*.

In this note, we illustrate the constraints that REH and NREH impose on macroeconomic analysis in the context of the present-value of stock prices. The model relates the price of a stock or a basket of stocks at every point in time, P_t, to the market's forecast of stocks' fundamental value, P_t^{F}:

$$P_t = \mathcal{F}_t^{\mathrm{M}}\left(P_t^{\mathrm{F}}|V_t\right) = \sum_{k=1}^{\infty} \rho^k \mathcal{F}_t^{\mathrm{M}}\left(D_{t+k}|V_t\right) \tag{1}$$

where P_t^{F} is defined as the present discounted value of future dividends:

$$P_t^{\mathrm{F}} = \sum_{k=1}^{\infty} \rho^k D_{t+k} \tag{2}$$

[10] NREH's account of fluctuations of asset prices around their fundamental values does not rely on a time-varying risk premium. Having ignored the importance of unanticipated structural change, REH theorists have appealed to such a premium in explaining the behavior in Figure 1. (References). The difficulty with such explanations is that contrary to what is required to explain fluctuations in Figure 1, empirical evidence points to pro- rather than counter cyclical movements of risk premiums. For a review of this evidence, see Frydman and Goldberg (2011, chapter 5) and Frydman et al (2014).

D_{t+1} denotes the dividends that are earned on holding stock between t and $t+1$, and we abstract from variation in the market's risk premium or interest rates by assuming a constant discount factor, ρ. $\mathcal{F}_t^{\text{M}}(\cdot|V_t)$ represents the market's time-t point forecast conditional on information about the factors that it considers relevant, denoted by V_t. The market's information set is usually thought to consist of fundamental factors. For example, we would expect that market participants would rely on recent trends in companies' earnings in forecasting dividends or monetary policy announcements in forecasting interest rates. The t subscript on the $\mathcal{F}_t^{\text{M}}(\cdot)$ operator recognizes that the strategy underpinning the markets' point forecast may change over time.[11]

The model implies that fundamental factors influence stock prices through the market's forecast of dividends. It predicts that as the market raises (lowers) its forecast of the discounted value of dividends, it bids up (down) stock prices. Changes in these forecasts arise from two sources: news about the factors that the market considers relevant and revisions in how the market interprets this news in forecasting dividends.

2.1 Formalizing an Economist's Understanding

In order to generate predictions about the co-movements between stock prices and fundamental factors, an economist must represent the market's forecast of dividends in terms of fundamental factors. He must also constrain *ex ante* how the structure of this representation might change in future periods, that is, how the market might revise its strategy for relating current information to future dividends. Both REH and NREH represent the market's forecasts in (1) by imposing consistency between its understanding of the dividend process and that of the economist. However, REH and NREH models impose very different constraints on structural change *ex ante.*

In order to highlight the difference between these constraints, we formalize an economist's understanding with a specification that recognizes that the dividend process does change over time:

$$D_{t+1} = d_{t+1} X_{t+1} \tag{3}$$

where X_{t+1} is a vector of fundamental factors that an economist uses to represent D_{t+1} at time $t+1$, and d_{t+1} is a parameter vector. Companies'

[11] In REH models that represent outcomes with a probability distribution *ex ante,* $\mathcal{F}^{\text{M}}(\cdot|v_t)$ is the mathematical expectation of the distribution. Models that are open to unanticipated structural change do not imply such probabilistic representations. In these models, $\mathcal{F}_t^{\text{M}}(\cdot|v_t)$ denotes the market's point forecast. For the properties of this operator and a comparison with the expectations opertaor, see Frydman and Goldberg (2015).

decisions to pay dividends depend in part on their recent and expected future profitability. In general, we would expect that dividends in each period would depend on many factors, from recent earnings and sales revenues to market projections and overall macroeconomic activity.

We characterize the X_t processes as a random walks with time-varying drifts, m_{t+1}:

$$X_{t+1} = m_{t+1} + X_t + \epsilon_{t+1} \tag{4}$$

where ϵ_{t+1} is a vector of random variables with mean zero. We represent structural change in the X_t processes by relating the drifts to current information on fundamental factors:

$$m_{t+1} = a_t + b_t X_t \tag{5}$$

where a_t and b_t are vectors of parameters.

The structure in (3)-(5) formalizes an economist's understanding of the process underpinning fundamental factors and dividends in all time periods. For example, at $t+1$, we have

$$X_{t+1} = a_t + (1 + b_t) X_t + \epsilon_{t+1} \tag{6}$$

$$D_{t+1} = d_{t+1} [a_t + (1 + b_t) X_t + \epsilon_{t+1}] \tag{7}$$

2.2 What Can Economists Really Know About the Future?

The specifications in (6) and (7) are open to the possibility that a different set of fundamental factors and/or different parameter values may be needed to represent the dividend process at different points in time.[12] As they stand, these specifications have no empirical content: they are compatible with any co-movements in time-series data. In order for the model to yield such content, an economist must constrain *ex ante*, at $t = 0$, the model's quantitative structure in future time periods. These constraints formalize what the economist believes he can know in advance about the unfolding of dividends in the future.

Both REH and NREH suppose that there are certain qualitative features that characterize the relationship between dividends and fundamental factors in all future time periods. For example, in this note, we assume for simplicity that X_t consists of one fundamental factor,

[12] Frydman et al. (2015) provide evidence that the set of fundamental factors that drives stock prices varies over time. For example, they find that oil prices underpin stock prices during some intervals of time and not during others.

companies' earnings. We suppose that higher earnings at a point time leads to both a higher level and mean change of dividends, that is,

$$d_t > 0 \text{ and } b_t > 0 \quad \text{for all } t \tag{8}$$

These qualitative constraints are consistent with myriad different quantitative structures at every point in time. They thus leave open which particular set of quantitative structures will characterize future dividends and earnings.

The qualitative constraints in (8) are not sufficient for the model to yield even qualitative predictions concerning the co-movements of stock prices, dividends, and earnings.[13] The economist must also constrain changes in structure across points of time. REH and NREH impose very different constraints on this change.

2.2.1 REH's Constraints on Structural Change

REH models rest on a striking premise regarding what economists and market participants can know *ex ante* about future structural change. REH supposes at $t = 0$ that an economist can specify in advance all *quantitative* structures that will be needed to account for how dividends and fundamental factors will actually unfold over the indefinite future. He does so by imposing constraints that specify exactly how these structures will change over time.

We illustrate these constraints by setting the parameters in (6) and (7) to be constants:[14]

$$d_t = d;\ a_t = a;\ b_t = b \tag{9}$$

The dependence of the drift m_{t+1} in (5) on X_t implies that the specifications of dividends and earnings in (3) and (4) undergo structural change in every period. However, by setting a_t and b_t to constants, an economist supposes at $t = 0$ that there is a probabilistic rule that can characterize how earnings will actually unfold in all future periods. By also setting d_t to a constant, the economist specifies ex ante all the quantitative structures that will be needed to account for future dividends, and therefore stocks' fundamental value, in terms of fundamental factors.[15] For example, at $t + 1$, we have:

$$D_{t+1} = \tilde{a} + \tilde{b}X_t + \epsilon_{t+1} \tag{10}$$

[13] For example, dividends and earnings would co-move positively (negatively) if every period the change in a_t was sufficiently large and of the same (opposite) sign as the change in X_t.

[14] See Barsky and Delong (1993) for an early example of such a formulation.

[15] This conclusion would also follow if we were to allow for the parameters in (3) and (5) to change over time according to a probabilistic rule, such as Markov switching. In such a case, the model would characterize the dividend process at $t + 1$, conditional

$$P_{t+1}^{\mathrm{F}} = \frac{\rho\left[\tilde{a} + (1-\rho)\,\tilde{b}a\right]}{(1-\rho)\left(1-\tilde{b}\rho\right)} + \frac{\rho\tilde{b}\,(1+b)}{1-\rho\tilde{b}}X_t + \nu_{t+1} \tag{11}$$

where $\tilde{a} = da$, $\tilde{b} = d\,(1+b)$, and ν_{t+1} depends on ϵ_{t+1}. Consequently, the model rules out *ex ante* that different sequences of structures than those implied by (9) and (11) will be needed to account for how dividends and stocks' fundamental value will actually unfold over time.

REH imposes consistency between an economist's prediction of future dividends and that of the market. An REH theorist, therefore, represents market participants' forecasting with his own probability distributions in (10) and (11). By doing so, he presumes that participants also believe that the quantitative relationship between dividends and earnings, and thus between stocks' fundamental value and earnings, can be fully foreseen in probabilistic terms. The resulting REH present-value model of stock prices is given by the following:

$$P_t = \frac{\tilde{a}\rho}{(1-\rho)\left(1-\tilde{b}\rho\right)} + \frac{\rho\tilde{b}}{1-\rho\tilde{b}}X_t \tag{12}$$

Having fully specified all structural change in advance, the model implies quantitative predictions concerning co-movements between stock prices and earnings:

$$cov\left(\Delta P_{t+1}, \Delta X_{t+1}\right) = \frac{\rho\tilde{b}}{1-\rho\tilde{b}}var_{\epsilon} > 0 \tag{13}$$

where $cov\left(\cdot\right)$ and var_{ϵ} denote an unconditional covariance and the variance of ϵ_t, respectively. According to this model, stock prices not only co-move positively with earnings, but the change in earnings has on average the same exact impact on stock prices in every time period [16]

2.2.2 The Irrationality of REH in Real-World Markets

In real-world markets, historical events such as a new Fed chairman or new CEO, sooner or later leads to quantitative change in the process underpinning dividends that an economist could not have fully specified in advance. Even if an REH model adequately characterized the unfolding of dividends prior to such unanticipated change, it would cease to do so afterwards. In Frydman and Goldberg (2015), we show that if the REH

on the values of the parameters d, a, and b and the information on fundamental factors at t.

[16] We are assuming that $\frac{\rho\tilde{b}}{1-\rho\tilde{b}} > 0$.

model continued to represent the market participants' forecasts after the point of structural change, participants' forecasting would involve obvious forecasting errors. As these errors accumulated, they would become obvious to detect. In real-world markets, therefore, REH represents decision-making by participants who are irrational: they ignore obvious forecast errors and thereby forego profit-opportunities.

Muth (1961, pp. 315-316) recognized early on that in order for an economist's model to serve as "the relevant economic theory" in representing the forecasts of participants who are rational and profit-seeking, it must "bear resemblance to the way the economy works." In ruling out, by design, all structural change that cannot be fully specified in advance, REH models ignore the central feature of real-world markets. Herein is the key reason REH models, though internally consistent, do *not* represent how rational, profit seeking participants make decisions in financial and other markets.

2.2.3 An Objectively Open Future

Our argument about the irrationality of REH models rests on recognizing the inherent limits to what we can know about the future. It seems uncontroversial that economists cannot anticipate, even in probabilistic terms, the exact sequence of historical events that will occur in the future, much less their quantitative impact on the process underpinning companies' profitability and thus their dividends. As Hayek (1974) pointed out in his Nobel lecture about "The Pretence of Knowledge,"

> Indeed, the chief point was already seen by those remarkable anticipators of modern economics, the Spanish schoolmen of the sixteenth century, who emphasized that what they called the mathematical price depended on so many particular circumstances that it could never be known to man but was known only to God.

Hayek is referring to our inability to understand "particular circumstances" underpinning outcomes even in hindsight. As we discussed in the overview, econometric studies of structural change based on *ex post* data bear out Hayek's compelling claim. These studies often find that market outcomes are characterized by very different quantitative structures and probabilistic switching rules across different subperiods of their samples. They also find that such change is often triggered by historical events that are at least in part unique. These findings imply that models that are estimated on the basis of *ex post* data will not provide an adequate *quantitative* account of future structural changes. Sooner or later unanticipated structural change that is not well approximated by in-sample estimates will occur.

NREH recognizes that because of unanticipated structural change, we cannot know the future in quantitative terms. As Karl Popper (1990, p18.) put it,

> Quite apart from the fact that we do not know the future, the future is objectively not fixed. The future is open: objectively open.

Consequently, any particular set of quantitative structures that an economist might select in advance will eventually become obsolete.

However, the process underpinning market outcomes might exhibit certain qualitative features that will persist into the indefinite future. NREH builds on this idea by imposing qualitative constraints *ex ante* on the structure of the economist's model at every point in time. It also imposes partly open constraints ex ante on how this structure might change during intervals of time. The bounds implied by these constraints leave partly open the exact magnitude of structural change in these intervals.

2.3 NREH

To illustrate NREH we continue to suppose *ex ante* that X_t consists of one fundamental factor, companies' earnings. We also continue to suppose that the economist can know in advance the *qualitative* relationships between earnings and the level and mean change of dividends by imposing the qualitative constraints in (8) *ex ante*. Consequently, we hypothesize that higher (lower) earnings at a point time leads to both a higher (lower) level and mean change of dividends in all future time periods.

Frydman and Goldberg (2015) show that these qualitative constraints imply a qualitative restriction on the relationship between stocks' fundamental value and current earnings at a point in time. This relationship can be expressed as follows:

$$P_t^{\mathrm{F}} = \hat{a}_t + \hat{b}_t X_t + \hat{\epsilon}_t \tag{14}$$

where the parameters $\hat{a}_t$ and $\hat{b}_t$ depend on the $\tilde{a}_{t+k}$ and $\tilde{b}_{t+k}$ parameters at every horizon k and $\hat{\epsilon}_t$ is a mean-zero error term that depends on ϵ_{t+k} at all k. Imposing the constraints in (8) *ex ante* implies that

$$\hat{b}_t > 0 \quad \text{for all } t \tag{15}$$

Internal consistency then implies the following NREH representations of the market's forecast of stocks' fundamental value and stock prices:

$$P_t = \mathcal{F}_t^{\mathrm{M}} \left(P_t^{\mathrm{F}} | X_t \right) = \alpha_t + \beta_t X_t \tag{16}$$

where

$$\beta_t > 0 \quad \text{for all } t \tag{17}$$

However, this qualitative restriction alone has no implication for the co-movement of P_t and X_t over time, since

$$\Delta P_{t+1} = \left[\Delta\alpha_{t+1} + \Delta\beta_{t+1}X_{t+1}\right] + \beta_t \Delta X_t \tag{18}$$

This equation shows that variation in stock prices depend on two sources: structural change effects—$\Delta\alpha_{t+1}$ and $\Delta\beta_{t+1}X_{t+1}$—and an informational effect—$\beta_t\Delta X_t$. Without *ex ante* constraints on structural change, the economist's specification yields no time series predictions concerning stock prices and fundamental factors.

Like REH, NREH makes use of the economist's understanding of how the dividend and thus the P_t^{F} process might change to represent that of market participants. However, NREH supposes that this change exhibits qualitative rather than quantitative regularities. Frydman and Goldberg (2015) suppose that this change is often moderate.

2.3.1 Intervals of Moderate Structural Change

An economist may consider many reasons why the relationship between stocks' fundamental value and current earnings might undergo quantitative structural change. For example, such shifts could stem from company level developments, including a new management team, the launch of a new product, mergers, and the opening of overseas markets. Developments in the broader economy could also affect the structure underpinning companies' prospects, from shifts in fiscal and monetary policy to regulatory reform, trade deals, and war.

Such developments, if they are sufficiently large, could lead to large quantitative changes in the dividend and P_t^{F} process. However, many of the developments that can lead to structural shifts in this process tend to remain largely unchanged or change very little for protracted intervals of time. During these intervals, we would expect moderate change in the relationship between stocks' fundamental value and current earnings.

Major shifts in management teams, policy, institutions, and other developments do eventually occur. We would thus expect that intervals of moderate change in the dividend and P_t^{F} process would be punctuated by moments when the quantitative structural change would be large. Such large shifts at the company level and in the macroeconomy can be anticipated only dimly, if at all. Consequently, no one can fully anticipate when intervals of moderate change in the macroeconomy might begin or end.

2.3.2 Partly-Open Constraints on Structural Change

Frydman and Goldberg (2015b) define moderate structural change to imply that its effects on stocks' fundamental value are smaller in magnitude than the informational effects. This definition implies the following partly open constraint on revisions of rational participants' forecasting strategies during intervals of moderate structural change:

$$\left|\Delta\alpha_{t+1} + \Delta\beta_{t+1}X_{t+1}\right| < \left|\beta_t \Delta X_{t+1}\right| \tag{19}$$

for t between $T_{j+1} - T_j + 1$ and all j, where interval j of moderate change begins at $t = T_j$ and ends at T_{j+1}. This condition constrains revisions in forecasting strategies to be moderate during the intervals of time in which structural change in the dividend and P_t^{F} process is moderate. As such, the economist hypothesizes at $t = 0$ that market participants on the whole will understand when this process undergoes moderate change and thus will revise their forecasting strategies in moderate ways during these periods.[17]

In sharp contrast to REH's constraints, the partly open constraint in (19) does not specify exactly how the quantitative structure that relates dividends to fundamental factors will change over adjacent periods during intervals of moderate change. This constraint is therefore open to unanticipated structural change, but only partly so. As a result, it yields time series implications for the co-movements of stock prices and earnings during the intervals of moderate change.

2.3.3 Qualitative and Contingent Predictions

In sharp contrast to its REH version, the NREH present-value model implies only qualitative and contingent predictions concerning co-movements between stock prices, dividends, and earnings. The model predicts that stock prices and dividends are positively related to earnings at every point in time, that is, $\beta_t > 0$ and $b_t > 0$. Moreover, with informational effects outweighing structural change effects during intervals of moderate change, the model predicts at $t = 0$ the following co-movements between stock prices, dividends, and earnings in those intervals:

$$sgn\left(\Delta P_{t+1}\Delta X_{t+1}\right) = sgn\left(\beta_t\right) > 0 \text{ and}$$
$$sgn\left(\Delta D_{t+1}\Delta X_{t+1}\right) = sgn\left(b_t\right) > 0 \tag{20}$$

[17] Frydman and Goldberg (2014) point out that this supposition is an implication of the premise that market participants' aggregate understanding of change encompasses that of the economist. REH models embody a particularly restrictive form of this premise: an economist's understanding is "essentially the same," in quantitative terms, as that of the market.

for t between $T_{j+1} - T_j + 1$ and all j.

The qualitative prediction that stock prices co-move positively with earnings during intervals of moderate structural change is borne out in Figure 1. The Figure shows that stock prices and earnings both underwent two major upswings (from 1993-2000 and 2002-07) and two major downswings (from 2000-02 and 2007-09). The NREH model characterizes each of these subperiods as a distinct interval of moderate structural change in which the mean change in earnings, m_t, was either positive or negative. The Figure suggests that in each of these subperiods, the relationship between stock prices and earnings, although positive, experienced quantitative structural change. For example, during the 1990s upswing, stock prices and earnings rose at roughly the same rate until the end of 1995, whereupon the rise in earnings flattened noticeably while the rise stock prices did not. Variables other than earnings no doubt underpinned price movements during this subperiod, which could account for this shift. But, historical developments from two presidential elections, shifts in monetary policy, and new applications of information technology suggest that moderate changes in the price process (that is, $\Delta\alpha_{t+1}$ and $\Delta\beta_{t+1}$) also underpinned stock price movements during the 1990s.

The NREH present-value model characterizes the transitions between major upswings and downswings in Figure 1 as brief periods in which large structural changes occurred, leading not only to shifts in m_t, but to switches in its algebraic sign. The Figure suggests that the positive relationship between stock prices and earnings also changed across these intervals of time; for example, the 1990's upswing was characterized by a tendency for stock prices to rise much faster than earnings, whereas the opposite was true during the 2000's upswing.

2.4 It is Better to be Roughly Right Than Precisely Wrong

NREH supposes that although the future is open, we can foresee certain qualitative features about the process underpinning market outcomes and how this process will unfold into the indefinite future. The NREH present-value model that we sketched in this note hypothesizes such qualitative knowledge: structural change in the process underpinning stock prices is moderate for protracted intervals of time and during these intervals, prices co-move positively with earnings. If we had included other fundamental factors in the model, it would yield additional qualitative and contingent predictions concerning co-movements in the data.

In Frydman and Goldberg (2015), we sketch how our NREH present-value model can account for the persistent price swings in Figure 2 in

terms of both rational forecasting and fundamental factors. The key to relating asset-price fluctuations to rationality and fundamentals is to jettison REH's premise that we can know the future in quantitative terms.

Dropping this premise is also the key to incorporating the insights of behavioral economists that psychological factors also play an important role in driving prices, but *without* the presumption that market participants are irrational. NREH implies that a rational participant understands that there are many ways to forecast and thus, cannot rely solely on statistical analysis or other calculations to ascertain which forecasting strategy he should use. Ultimately, he is guided by the confidence that he has in choosing one strategy over others to relate available information on fundamental factors to future outcomes. Intuition and emotions (such as optimism and fear) inevitably also play a role in how a rational, profit-seeking participant *chooses* his forecasting strategy and when and how to revise it.

Consequently, the dualism between rational forecasting and the importance of psychological factors is an artifact of REH's premise that we can know the future in quantitative terms. Psychological factors underpin market participants' forecasting not because they are irrational, but because they must rely on these factors to interpret how fundamental factors will drive future market outcomes. In Frydman and Goldberg (2015b), we present evidence that psychological factors underpin daily stock price movements in precisely this way.[18]

Taken as a whole, our research shows that we can learn more about asset markets if we search for only qualitative knowledge. As Keynes put it, "it is better to be roughly right than precisely wrong."[19]

[18] The evidence is based on a novel data due to Mangee (2011) that converts textual information on daily stock price movements from *Bloomberg News* into numerical data.

[19] Quote attributed to John M. Keynes by Alan Greenspan (1997, p. 372).

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