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The Contingent Expectations Hypothesis:

Rationality and Contingent Knowledge in Macroeconomics and Finance Theory[*]

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1 Introduction: The Choice for Macroeconomics

For macroeconomists, an individual is rational if she uses her understanding of the way the economy works in making decisions that do not conflict with her objectives. In this paper, we reconsider how macroeconomists can build models that are compatible with rational decision-making. We show that, by design, the Rational Expectations Hypothesis (REH) is an abstraction of rational decision-making in a world in which individuals' knowledge about the process underlying market outcomes does not grow over time. This conclusion concerning REH's limited domain of applicability leads us to propose the contingent expectations hypothesis (CEH) for modeling rational forecasting in markets in which outcomes are driven in part by the growth of participants' knowledge. CEH rests on Karl Popper's insights concerning the importance of recognizing the growth of knowledge for developing empirically relevant economic models.[1] Our hypothesis also rests on key insights by John Muth and Robert Lucas that led to REH: an economist's own model can be used to build more rationality into his analysis, and compatibility with rational decision-making requires that the model be internally coherent.

We show how CEH provides a way to synthesize REH's focus on the importance of fundamental factors in underpinning rational forecasting with the two other major advances in macroeconomics over the last four decades: Phelps's *et al* (1970) research program of basing aggregate relationships on micro-foundations that accord an autonomous role to participants' expectations, and behavioral economists' use of empirical observation in representing individuals' decision-making. By recognizing the importance of the growth of participants' knowledge for how market outcomes unfold over time, CEH-based analysis can incorporate these approaches, which are usually thought to be incompatible with profit-seeking and rational behavior, into macroeconomic models that are compatible with rational decision-making.

The importance of constructing such models was the impetus for the rational-expectations revolution. In proposing (REH), Muth (1961, p. 315) persuasively argued that "a systematic theory of fluctuations in markets or in the economy" requires that models of aggregate outcomes "include an explanation of the way expectations are formed." The difficulty in relating

[1] Popper's (1946, 1957, 1983) insights on the growth of knowledge have had a profound impact on our understanding of the scientific process and thinking about change in societies. Our reliance on those insights builds on George Soros's use of them. Soros's (1987) analysis was based on the idea of "fallibility": every understanding of markets is necessarily contingent, eventually becoming inadequate and requiring revision.

"the way expectations are formed" to individuals' understanding of the economy is that there are many ways to understand the process that underpins outcomes in real-world markets – and many ways to understand when and how this process might change. Moreover, market outcomes are driven by participants' combined buy and sell decisions, which are typically related in macroeconomic models to the aggregate forecast. Any formal representation of the understanding – the knowledge – that underpins the market's forecast must therefore be a bold abstraction.

Muth's (1961) striking insight was that an economist could use his own model to represent how the market – the aggregate of participants – understands how the economy works and how it forecasts outcomes. REH formalized this idea: market participants' "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory" (p. 316).

Although imposing consistency within a macroeconomic model is necessary, it is far from sufficient to render a model "relevant" for representing how rational participants understand and forecast economic outcomes over long stretches of time. To be sure, the ultimate test of any model's relevance is its empirical adequacy in accounting for the observed regularities in time-series data. On this score, the performance of REH models has been less than stellar; they have encountered widespread empirical difficulties in many markets. But REH can be used in myriad models. Thus, rejection of one REH model does not preclude the possibility that another, either existing or yet to be invented, might be empirically adequate and thus could serve as a basis for representing rational forecasting.

Indeed, macroeconomists continue to expend enormous resources and talent in this search. In this paper, we advance a theoretical argument that provides an a priori criterion for deciding whether we should continue to rely on REH models, or redirect our research agenda to developing an alternative class of models on which to base our representations of rational forecasting.

2 Overview

Our argument makes use of the observation that all macroeconomic models that can serve as a basis for representing rational forecasting can be divided into two mutually exclusive classes. Models in both classes relate outcomes to a set of causal factors and characterize the process that governs those factors. This "structure" expresses an economist's understanding of the process driving aggregate outcomes, including how market participants forecast.

Model structures in the two classes may share specifications of participants' preferences and other components. But, as time passes, the process that underpins economic outcomes may change, at least intermittently, implying that distinct structures may be required to represent this process during different stretches of time.

The two classes of models formalize sharply different conceptions of such change. One class represents change as *determinate*: conditional on their structure at any point in time, these models specify in advance all potential structures that might represent the process driving outcomes at any other point in time.[2] The other class leaves its models *partly open* to unanticipated *structural* change: these models constrain change in their structures over time; but, conditional on any one of the causal structures that is implied by a model at a given point in time, they do not specify in advance the *exact* structures that may be needed to represent the market process at any other point in time.

In conceiving REH, Muth (1961, p. 315-316) emphasized that "the way expectations [are formed] would change when. . . .the structure of the system is changed." Muth's idea was that an economist's model could serve as the basis for representing changes in "the structure of the system" as well as market participants' understanding of such changes. But, because an economist's model provides "informed predictions of future events" (p. 316), he must choose which of the two classes of models he considers relevant for representing change. By choosing a determinate model as the basis for representing individuals' understanding of how the "structure of the system is changed," later theorists in effect adopted a particular conception of change on which to base their representations of rational forecasting. As Lucas (1995, p. 255; 2001, p. 13) pointed out, given this choice, acceptance of REH models as the only valid way to represent rational forecasting follows on logical grounds.[3]

[2] As we show in Frydman and Goldberg (2007), the class of determinate models includes standard REH models, in which the causal structure is constrained to be time-invariant. It also includes REH bubble and multiple-equilibrium models, as well as behavioral-finance models. In section 2, we provide a formal definition of a model's causal structure, as well as an example of a determinate model that is typical of macroeconomic and finance theory. In Frydman and Goldberg (2007) and our subsequent work, we refer to models that fully prespecify change in probabilistic terms as "fully predetermined." In Frydman and Goldberg (2013a), we pointed out that all such models are determinate. Fully predetermined models specify fully in advance not only all potential structures, but also when and how changes between structures might occur.

[3] The argument that standard REH models follow on logical grounds underpins the debate between Krugman (2009) and Cochrane (2009) about the rationality of markets. In Frydman and Goldberg (2011, chapter 1), we suggested that the real problem is that both Cochrane and Krugman have relied on the same REH-based conception of rationality. For a mathematical example and further discussion, see section 3 below and Frydman and Goldberg (2013a)

By choosing a conception of change to underpin his model, an economist selects the kind and duration of the regularities that he supposes his model is capable of explaining. Constraining structural change to be determinate presumes that the relevant regularities for modeling rational forecasting are determinate. Thus, REH models imply regularities that relate in exact probabilistic terms how outcomes and causal variables co-move over time. These models also imply that these regularities last for long stretches of time, spanning decades. REH models recognize that to be rational in such a world, a profit-seeking individual's forecasts of outcomes must be consistent with the hypothesized determinate regularities.

By contrast, partly open models recognize the importance of the growth of knowledge, and thus remain open to unanticipated change in the process underpinning market outcomes. These models do not imply that outcomes and causal variables co-move in a fixed way. Nevertheless, there are stretches of time during which the co-movements in these variables are characterized by distinct qualitative relationships, for example, that they co-move positively. These models also imply that, sooner or later, structural change will occur, and that it could bring different qualitative relationships or a different set of variables to the fore in the process driving outcomes.[4] By supposing that these stretches of time last long enough to be discernible by statistical or other methods, CEH recognizes that, in order to be rational, profit-seeking individuals' forecasts must be consistent with the hypothesized qualitative and contingent regularities.

Every macroeconomic and finance model that aims to represent rational forecasting should be seen as an abstraction of decision-making in only one of two "worlds." Determinate models are abstractions of a world in which participants' understanding of the economy does not grow, while models that are partly open are abstractions of a world in which such knowledge does grow. We show in section 3 that this conclusion follows from Popper's (1957, xii) proposition: "If there is such a thing as growing human knowledge, we cannot anticipate today what we shall only know tomorrow."

REH models are necessarily bold abstractions of the many ways that participants can understand how the economy works. But, whatever this diversity of knowledge might be, approximating it with a determinate model represents it with a common feature: individuals believe that they can anticipate today how they will understand the process driving outcomes in the

[4] Our example in section 5 implies stretches of time during which the economy's inflation rate co-moves positively with a real interest rate. Our example also implies that changes in tax policy might cause a shift in this relationship from the positive the negative co-movement between these variable. The model leaves open the exact timing and nature of such change.

future. REH models should be seen, therefore, as abstractions of rational forecasting in "markets" in which the knowledge that underpins the aggregate forecast does not grow.

This conclusion leaves open the question of whether REH models could nonetheless serve as representations of rational decision-making in real-world markets. Proponents have rightly argued that, because REH models are bold abstractions, viewing them as literal descriptions of how profit-seeking individuals understand and forecast the market process is "misleading, because [it] make[s] rational expectations sound less restrictive and more behavioral in its foundations than it really is" (Sargent, 1993, p. 21). Nonetheless, Sargent viewed REH as a way to represent formally rational forecasting in real-world markets. Like many others, he regarded REH models as abstract approximations of "the outcome of [an admittedly very complex market] process in which people have optimally *chosen* their perceptions" (Sargent, 1993, p. 7).

In section 4, we examine whether REH models could approximate rational decision-making in markets in which knowledge grows. Our example is highly stylized, but we follow Muth, Lucas, Sargent, and most other macroeconomists by representing the growth of knowledge as an unanticipated change in the structure of an economic model.[5] We show that this definition means that REH models do not adequately approximate decision-making by "people [who] have optimally chosen their perceptions": in markets in which knowledge grows, these models represent forecasting by individuals who forego *obvious* profit opportunities.

REH models have profoundly shaped our thinking about markets, and have altered how we regulate them and conduct economic policy. Although REH models are incompatible with rationality in markets in which knowledge grows, their implications may nonetheless have a basis in how rational participants make decisions in such markets. But, in order to examine which of the REH-based implications can be related to rationality, we need to develop an analog for REH that is relevant for building models for markets in which knowledge grows.

In section 5, we propose CEH as such an analog. Our hypothesis rests on

[5] Some may be tempted to represent the growth of knowledge with probabilistic uncertainty, that is, with independently distributed mean-zero random disturbances. However, relying on such shocks to represent the growth of knowledge would undermine REH policy analysis, which rests largely on the premise that change in policy rules, not just random disturbances, alters participants' understanding of the economy in terms of the set of causal factors (Lucas, 1976). Portraying the growth of knowledge as a stochastic disturbance is also contrary to behavioral-finance models, which represent changes in forecasting strategies as changes in a model's structure. For examples, see Frankel and Froot (1987), Brunnermeier (2001), and LeBaron (2013), and references therein.

two principles. The first, which we call the *principle of contingent knowledge*, rules out determinate models as candidates for building formal accounts that are compatible with rational decision-making in markets in which knowledge grows. This principle follows logically from our conclusion that determinate models are inadequate in accounting for how the process driving outcomes in these markets changes. Consequently, relying on any model in the determinate class to represent forecasting, regardless of whether it employs REH, is to portray decision-making by individuals who do not understand that they forecast in a world in which knowledge about the economy grows.

An influential recent attempt to represent rational forecasting in determinate models without REH makes use of least squares and other algorithmic learning rules. In explaining the use of such rules, Evans and Honkapohja (2013, p. 68) point out that "economists, when they forecast... usually do so using time-series econometric techniques." This leads them to argue that "economic agents should be about as smart as (good) economists," and to propose the algorithmic learning approach as a "rational foundation for macroeconomic and finance" models (p. 69). But, while these models are bold abstractions of decision-making by individuals whose "knowledge is quite imperfect" (p. 69), they nonetheless assume that the knowledge underpinning the market's forecast does not grow.

Our arguments point to the flaw in attempts to use determinate models to represent how individuals cope with imperfect knowledge: the growth of knowledge that underpins the market process applies to economists as well.[6] Thus, in order to represent how rational individuals cope with imperfect knowledge, economists should recognize their own imperfect knowledge, not just that of market participants. The principle of contingent knowledge implies that doing so would require reliance on partly open, rather than determinate, models.[7]

Although the principle of contingent knowledge rules out determinate

[6] In Frydman and Goldberg (2007, 2011), we argue that this claim follows on empirical grounds from the assumption that market participants are profit-seeking. The arguments presented here provide the theoretical rationale for this claim.

[7] Using determinate models to represent forecasting by individuals who cope with imperfect knowledge is becoming quite popular. Evans and Honkophja (2013) argue that these models enable macroeconomists to move away from REH and yet preserve some relation to rational decision-making in real-world markets. To be sure, representing market participants' forecasting with a learning algorithm implies that the structure of an economist's model of aggregate outcomes changes over time. But specifying structural change with a determinate rule presumes that participants understand today exactly how they will "learn" about the process driving outcomes in the future. For a discussion along these lines of algorithmic learning and related models, such as eductive games in Guesnerie (2005), see Frydman and Phelps (2013).

models, models that are consistent with it include those that do not impose any constraints on structural change. These completely open models do not have any implications for time-series data and thus, ipso facto, cannot represent economists' or market participants' understanding of how the process underpinning market outcomes changes over time. This uncontroversial point implies that when applying the principle of contingent knowledge in models that could represent rational forecasting, macroeconomists should exclude not just determinate models, but also models that are completely open.

Implicit in this argument is the observation that an open system "is not the opposite of a closed system, since there is a range of possibilities for openness, depending on which conditions are not met and to what degree.…Deviating from a closed system does not mean abandoning theory or formal models" (Dow, 2013, p.122). Indeed, partial opening of a macroeconomic model entails imposing constraints on structural change that stop short of specifying in advance the exact structures that may be needed to represent how the market process and participants' understanding of it unfold over time.

In Frydman and Goldberg (2007), we proposed imperfect knowledge economics (IKE) as an approach to building models that are partly open.[8] IKE represents how individual decision-making and other features of the social context, including policymaking, unfold over time by imposing qualitative and contingent restrictions on change in a model's structure. These IKE constraints are less stringent than those that determinate models employ to represent change in the process underlying market outcomes. This partial openness is required for any model to be an abstraction of rational forecasting in markets in which knowledge grows. We provide a formal example of IKE constraints in section 6.

By design, all IKE models are consistent with the principle of contingent knowledge. Remarkably, Muth's and Lucas's insight concerning the importance of a model's internal consistency implies that not all IKE models are compatible with rational decision-making. And yet, because IKE models do imply regularities in time-series data, they, too, must represent how rational participants understand the market process in ways that are compatible with those regularities.

For the purposes of CEH, we refer to this analog for partly open models of REH's imposition of internal consistency as the *principle of internal*

[8] Our development of imperfect knowledge economics as a way to construct partly open models, and IKE's application to the analysis of asset markets, builds on Soros's (1987) original framework for understanding fluctuations in those markets. Soros (2009, 2012) makes use of this premise to explain the global financial crisis that began in 2008 and the euro-zone crisis.

coherence. Its rationale is analogous to the argument used by Lucas (1995, 2001) to justify REH in determinate models: representations that do not satisfy this principle are incompatible with the assumption that individuals' decision-making does not conflict with their objectives, such as maximizing utility or profits.

Like internal consistency in REH models, we show in section 6 how the principle of internal coherence connects an economist's representation of forecasting to the specifications of his models' other components, and how this connection restricts changes in a model's structure over time. Internal coherence plays a key role in opening macroeconomic models to change in ways that cannot be fully foreseen by anyone, yet it does so in IKE models without abandoning either time-series implications or rational decision-making. Moreover, as IKE models stop short of specifying structural change exactly, imposing internal coherence necessarily leaves them open to representing market participants' forecasting as partly autonomous, that is, partly independent of a model's other components, such as preferences or constraints. However, this principle does not leave CEH-based models completely open: it does constrain revisions in how participants forecast in ways that imply empirical regularities.

IKE thus provides a way to advance the research agenda that was originated by the Phelps et al (1970) volume.[9] But the insight that participants' expectations are an important autonomous factor in driving outcomes was formalized by the contributors to the Phelps volume within the context of determinate models. When the REH revolution made clear that these internally inconsistent models could not serve as a basis for representing rational decision-making, expectations' autonomous role was jettisoned from macroeconomic models.[10] The argument here implies that the class of determinate models, not autonomous expectations, should have been abandoned.

The behavioral school resurrected the importance of autonomous expectations for macroeconomic analysis. This enabled researchers to incorporate into their models important empirical findings concerning how individuals actually behave in market settings.[11] But, despite its emphasis on the need to incorporate more realism into economic models, the behavioral-finance approach has formalized its empirical findings with determinate models.[12]

[9] To paraphrase Sargent (2005), in IKE models "people's beliefs are in part inputs. They are not just solely outcomes of our theorizing."

[10] See Frydman and Phelps (2013) for an extensive discussion of this important break in the direction of macroeconomic research in the early 1970's.

[11] See Shleifer (2000) and Barberis and Thaler (2001) and references therein.

[12] For a notable exception, see Akerlof and Shiller (2009), who rely on a narrative mode of analysis, and thus *ipso facto* avoid representing behavioral insights with determinate

Behavioral researchers recognize that their models embody internal inconsistencies. But their research program rests on the premise that relating our micro-foundations to empirical observations is crucial for remedying REH models' empirical failures.

What made abandoning internal consistency seem plausible is that departures from REH-based rationality seemed to imply correlations in time-series data that suggested that individuals forego profit opportunities after all. This led behavioral economists to construct models in which market participants were obviously irrational – for example, they systematically over- and underreact to news in forecasting asset prices.[13] But proponents of REH-based rationality argued that these correlations would become obsolete with the passage of time. As Fama has observed, "apparent over-reaction to information is about as common as under-reaction, and post-event continuation of pre-event abnormal returns is about as frequent as post-event reversals" (Fama, 1998, p. 283).

The argument here implies that for our models to have the potential to explain regularities in time-series data, they must be partly open and accord participants' expectations an autonomous role. Doing so implies that we will need psychological considerations and other factors that behavioral economists have emphasized in order to understand individual decision-making and aggregate outcomes. As Keynes clearly saw early on, recognizing the role of psychology does not necessarily imply irrationality:

> We are merely reminding ourselves that our rational selves [are] choosing between alternatives as best as we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance. [Keynes, 1936, p. 136, emphasis added]

In section 6, we show how CEH enables us to incorporate psychological considerations into IKE models in ways that are compatible with rational decision-making in real-world markets where knowledge clearly grows.

3 Rationality in a Determinate World

There are many ways to understand the process driving market outcomes. Muth's (1961, p. 316) fundamental insight was that an economist's own model could serve as a basis for representing participants' understanding

models. For further discussion, see Frydman and Goldberg (2011, Chapter 1).

[13] See Barberis, Shleifer, and Vishny (1998) and Gourinshas (2004), and references therein.

of this process in terms of a set of causal factors. In order to formalize this insight, he chose a determinate model and the rational expectations hypothesis to represent his own and market participants' understanding of how market outcomes unfold over time.[14]

Muth viewed REH as a "purely descriptive hypothesis" about the aggregate of market participants' understanding of the process driving outcomes. However, in the early 1970s, Lucas recognized that once an economist chooses a determinate model to represent this process, REH follows as the only valid way to represent how a profit-seeking rational individual understands it and forecasts its outcomes[15] Beyond rendering determinate models compatible with rational decision-making, the embrace of REH was also seen as essential to these models' ability to explain co-movements among aggregate outcomes and causal variables.

By design, non-REH models represent participants' forecasting as at least partly autonomous of their preferences and the model's other components. As we illustrate shortly, Lucas (1972b, 1973) and Sargent (1981) pointed out that determinate models with autonomous forecasting could at best account adequately for co-movements among aggregate outcomes and causal factors during a limited period of time. Sooner or later, a number of participants would recognize that their forecasting strategies implied systematic forecast errors and revise them. But, as soon as such revisions altered the market's forecast, the economist's model would no longer be "the relevant economic theory" of time-series regularities.

To be sure, REH models do rid determinate models of the presumption that market participants forego obvious profit opportunities. Consequently, they do imply longer-term regularities. But, as we illustrate formally in the next two sections, REH models are abstractions of rational decision making and its implications for aggregate outcomes only in markets in which participants' understanding of this market process – the knowledge that underpins the market's forecast – does not grow over time.

[14] Prior to REH, economists often portrayed forecast revisions adaptive expectations (Cagan, 1956, Friedman, 1957). Muth (1961, pp. 315-316) argued that, because they do not bear "resemblance to the way the economy works," such fixed error-correcting rules "do not assume enough rationality," which, he pointed out, "also applies to dynamic theories in which expectations do not explicitly appear" (citing the theories of competitive equilibrium in Arrow and Hurwicz (1958) and Arrow et al (1959) as examples).

[15] For early uses of REH-based micro-foundations, see Lucas (1972a).

3.1 REH: Choosing to Maintain Determinate Models

We begin with a formal illustration of how REH models are rendered compatible with rational decision-making by abstracting from any change in the aggregate of participants' understanding of the process driving market outcomes. To this end, we define participants' knowledge that underpins the market's forecast in the context of a stripped-down version of a model that is typical in contemporary macroeconomics and finance theory The vast majority of such models are not only determinate, but time-invariant. The following semi-reduced form represents an aggregate outcome, say, the market price:

$$P_t = bX_t + c\hat{P}_{t|t+1} \quad \text{for all } t \tag{1}$$

where $\hat{P}_{t|t+1}$ is the market's forecast formed at t of the price at $t+1$, (b, c) is a vector of parameters, and X_t is a set of causal factors.[16]

Formal accounts of how individuals understand the economy relate market outcomes to a set of causal factors, which portray the information ("facts") that market participants consider relevant. A typical representation of the knowledge that underpins the market's forecast can be written as

$$\hat{P}_{t|t+1} = \beta Z_t \quad \text{for all } t \tag{2}$$

where β is a vector of parameters. and Z_t characterizes the union of information sets used by market participants.

To simplify the presentation, we portray these causal factors with random walks:

$$X_t = \mu^{\mathrm{X}} + X_{t-1} + \varepsilon_t^{\mathrm{X}} \quad \text{for all } t \tag{3}$$

$$Z_t = \mu^{\mathrm{Z}} + Z_{t-1} + \varepsilon_t^{\mathrm{Z}} \quad \text{for all } t \tag{4}$$

where (μ^x, μ^z) are drifts and $(\varepsilon_t^x, \varepsilon_t^z)$ is vector of independently distributed disturbances ("the news"), which is characterized by an invariant probability distribution with the mean zero and some finite variance-covariance matrix.

Functional forms of equations (1) and (2), together with specifications of the processes governing the movements in the causal factors in (3) and (4), constitute the causal structure of the model. The model's structure at any point in time expresses an economist's understanding of the causal factors that he considers relevant, and the process by which outcomes – the

[16] Equation (1) has been used to model the money, currency, and equity markets, where X_t would include money and income in the first two cases and dividends in the latter. In all of these contexts, equation (1) can be derived from explicit microfoundations. For an excellent treatment of this issue, see Obstfeld and Rogoff (1996).

price and the market's forecast – are related to those factors. Like a vast majority of macroeconomic and finance models, the structure of our example is constrained not to change over time. Such models disregard the possibility that, as time passes, different structures may be needed to represent change in how the causal factors, the process driving the price, and market participants' understanding of this process unfold over time.

In general, the representation in (2) is autonomous of the other components of the model. Any such representation implies the following forecast errors:

$$FE_t^{\mathrm{AE}} = P_{t+1} - \hat{P}_{t|t+1} = \mu + bX_t + (c-1)\,\beta Z_t + \varepsilon_t \tag{5}$$

where, $\mu = b\mu^x + c\beta\mu^z$ and $\varepsilon_t = b\varepsilon_t^x + c\beta\varepsilon_t^z$.

Expression (5) represents the forecast errors implied by participants' autonomous forecasting as systematic and easily detectable: they have a non-zero mean and are correlated with Z_t, which represents the information used by participants in forming their forecasts. Thus, representing these forecasts as autonomous presumes that market participants repeatedly forego obvious profit opportunities. Hence Lucas's (2001, p. 13) warning: "if your theory reveals profit opportunities, you have the wrong theory."

This conclusion has posed a stark choice for the direction of any theoretical approach that would rid macroeconomic and finance models of gross irrationality and thereby open the possibility that they might adequately explain regularities in time-series data. Such an approach either could maintain determinate models to represent outcomes and the market's forecast, and jettison representing that forecast as autonomous, or it could maintain an autonomous role for participants' forecasting and jettison determinate models. In the event, Lucas and the vast majority of macroeconomists have steadfastly maintained determinate models as the way to understand the economy.[17]

3.2 REH: Abstracting from the Growth of Knowledge

Once a determinate model has been chosen to represent outcomes, REH has followed on logical grounds as the only valid way to represent how profit-seeking market participants understand the structure of the economy and use this knowledge to forecast outcomes. Indeed, imposing REH rids any determinate model of the presumption that market participants disregard

[17]In his Nobel lecture, Lucas (1995) discusses how the internal inconsistency of the determinate, so-called Keynesian, econometric models of the 1960s led him to assert that they are "the wrong theory."

endlessly systematic and easily detectable forecast errors. The hypothesis is not only compatible with profit-seeking, but it also relates the way in which market participants forecast market outcomes to their understanding of the process driving these outcomes.

A structure of an economist's REH model represents participants' understanding of the process driving outcomes by setting the market's forecast to equal the mathematical expectation of the price process, P_t^{EM} in (1), conditional on time-t information, I_t:

$$\hat{P}^{\text{RE}}_{t|t+1} = E\left[P_t^{\text{EM}} | I_t\right] \tag{6}$$

Applying (6) in (1) results in the following representation of the process driving the price and the market's forecast:[18]

$$P_t^{\text{RE}} = \frac{bc}{(1-c)^2}\mu^x + \frac{b}{1-c}X_t \text{...for all } t \tag{7}$$

$$\hat{P}^{\text{RE}}_{t|t+1} = \frac{bc}{(1-c)^2}\mu^x + \frac{b}{1-c}X_t \quad \text{for all } t \tag{8}$$

Thus, by imposing REH in a time-invariant model, an economist represents rational decision-making and its implications for aggregate outcomes in markets in which participants' knowledge concerning the process that underpins the market's forecast remains unchanged over time.

3.2.1 Allowing for Fully Anticipated Change in Structure

Some REH models do recognize change explicitly, but they represent it as determinate. By design, REH models specify participants' forecasting endogenously. Thus, in order to allow for change structure representing the processes driving aggregate outcomes and the market's forecast, an REH theorist must move away from time-invariant specifications of the non-expectational components of his model. Such modifications have sometimes allowed for stochastic preferences (for example, Barberis et al. 2001). However, following Lucas (1976), revisions of participants' forecasting strategies are usually modeled as arising from changes in the stochastic process that characterizes government policymaking.

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[18] In general, the parameters and causal factors in the representations for the price process in (1) depend on the specification of non-expectational components of the model's microfoundations, that is, market participants' preferences and the constraints that they face.

Hamilton (1988, 1994) developed a particularly influential class of determinate models that model change as determinate. A model in this class specifies a set of time-invariant REH models, typically two, to represent outcomes during different time periods. It fully specifies all model structures that would be needed to represent adequately how the process driving market outcomes, and how participants understanding of this process, might unfold over time. The Hamilton model also specifies the timing of these changes with a probabilistic Markov switching rule. We now show that, like their time-invariant counterparts, such determinate models of change also abstract from the growth of knowledge in representing how individuals make decisions and how aggregate outcomes unfold over time.

We consider the case in which the vector of causal factors, X_t, consists of only one variable, say, money supply. We set the drift in μ^x of the process for X_t in (1) to undergo a one-time shift. The simplest way to represent this change as determinate is to suppose that it is known that at t and earlier, the drift is equal to μ_1^x, and that at $t+1$ the drift will shift permanently to μ_2^x .and remain at the new level thereafter.

As is often the case in REH models that allow for change, we assume that the switch in the policy rule is the only way in which the model's structure could change over time. Our determinate restriction on change in this structure takes a particularly simple form:

$$\bar{A} = \mu_2^x - \mu_1^x \tag{9}$$

With this fully anticipated change in the process governing the policy variable at $t+1$, REH representations for the price process and the market's forecast take the following form:

$$P_{t+\tau}^{RE} = \frac{bc}{(1-c)^2}\left(\mu_1^x + \bar{A} + D\,(c-1)\,\bar{A}\right) + \frac{b}{1-c}X_{t+\tau} \quad \text{for all } \tau \tag{10}$$

$$\hat{P}_{t+\tau|t+\tau+1}^{RE} = \frac{b}{(1-c)^2}\left(\mu_1^x + \bar{A} + D\,(c-1)\,\bar{A}\right) + \frac{b}{1-c}X_{t+\tau} \quad \text{for all } \tau \tag{11}$$

where D is a dummy variable,

$$\begin{aligned} D &= 1 \text{ for } \tau = 0 \text{ and} \\ D &= 0 \text{ for all } \tau > 0 \end{aligned}$$

Specification in (11) represents the aggregate understanding of market participants that underpins the market's forecast in terms of a set of causal factors,

X_t, in all time periods. However, although the model allows for change in market participants' understanding, it specifies in advance that the change will occur at $t + 1$. The model also fully specifies the structure in (11) for $\tau \geq 1$, which is assumed to represent adequately the post-change market's forecast.

This specification thus represents decision-making in markets in which participants can fully anticipate today what they will know in the future about the price process. Because the model fully pre-specifies the change in market participants' understanding, it assumes that individuals never revise it in ways that they could not anticipate.

As Popper (1957, 1982) showed, even if an economist's model allows participants' understanding of the price process to change, ruling out unanticipated change is tantamount to representing decision-making in markets in which knowledge does not grow. Popper's proposition, slightly paraphrased, can be restated as follows:

> If there is such a thing as growing human knowledge, then no individual, such as an economist or a market participant, or group of individuals, such as market participants in the aggregate, can anticipate today what they shall only know tomorrow. (Popper, 1957, xii)

4 Growth of Knowledge and the Irrationality of Rational Expectations

REH models, we have shown, represent decision-making in markets in which participants' understanding of the process driving outcomes – the knowledge that underpins the market's forecast – does not grow over time. This leaves open the question of whether these models are nonetheless relevant in modeling outcomes in which participants' knowledge might change in ways that they could not have anticipated in advance. After all, as their proponents rightly argue, REH models are not literal descriptions, but bold abstractions. However, in this section, we show that REH models do not adequately represent how profit-seeking participants make decisions in markets in which knowledge grows. Indeed, REH models represent decisions by grossly irrational individuals who forego obvious profit opportunities.

This conclusion follows from the way in which macroeconomic models, whether REH or IKE, formalize an economist's and market participants' understanding of how the economy works. Formal economic theory represents this knowledge with a causal structure that relates outcomes – the price and the market forecast – to a set of causal factors. Given Popper's proposition, in order to represent the growth of knowledge that underpins the process driving aggregate outcomes, we would have to suppose that sooner or later a different causal structure – one that no one could have fully anticipated – would be needed to represent an economist's or market participants' knowledge.

Thus, any formal representation of decision-making in markets in which knowledge grows must allow for an unanticipated change either in the structure of a semi-reduced form of an economist's model, or in the model's representation of the market's forecast. Thus, recognizing the growth of knowledge in an economist's model necessarily implies recognizing REH models' inadequacy in representing rational decision-making and longer-term regularities in time-series data. Simply put, even if we were to suppose that an REH model might have adequately accounted for aggregate outcomes and the market's forecast during some stretch of time, the growth of knowledge would sooner or later render the model inconsistent with the assumption that market participants are profit-seeking.

4.1 A Stylized Example

As we discussed in connection with expression (4) for non-REH forecast errors in determinate models, Lucas has argued that according expectations an autonomous role in such models presumes that individuals forego obvious profit opportunities. Remarkably, once we open our models to allow for growth in the knowledge that underpins the market's forecast, Lucas's reasoning leads us to the opposite conclusion: REH models represent decision-making by irrational individuals who forego obvious profit opportunities.

To illustrate our argument formally, we suppose that the time-invariant REH model in (7) and (8) is relevant for representing the price process and the market forecast during some time period, $[T_1, T]$. This supposition follows directly from REH models' core premise: their semi-reduced and reduced forms are presumed to be relevant for representing outcomes in all time periods. To be sure, we recognize that, in imposing REH in a time-invariant model, an economist views it as an adequate approximation of the price process and the market forecast, rather than an assertion that the actual processes underpinning them remain exactly the same with the passage of time.

The adequacy of a determinate model can be judged in terms of the difference between the time-series of the actual outcomes and the model's representation of these outcomes. Typically, the model is considered to be adequate for some time, say, between T_1 and T, if its error at each point in time during that period does not depend on its past values or on the current and past values of causal factors included in the model's representations of outcomes[19]

Denoting this information set by I_t and the model's error by FE_t^{M}, we formally express the condition for the adequacy of the model's representation of prices during $[T_1, T]$, as follows:[20]

$$E\left[FE_t^{\text{M}}|I_t\right] = E\left[\left(P_t - P_t^{\text{M}}\right)|I_t\right] = 0 \quad \text{for all } t \in [T_1, T] \tag{12}$$

where, P_t and P_t^{M} denote, respectively, the actual price and its representation in the context of an economist's model.

In imposing REH, an economist hypothesizes that his model is adequate in the sense of (12) for all time periods. If this were the case the model would be adequate for at least a period of time that we denote by $[T_1, T]]$.Consequently, the following REH representations would adequately account for outcomes during the period $[T_1, T]$

$$P_t^{\text{RE}} = \frac{bc}{(1-c)^2}\mu^x + \frac{b}{1-c}X_t.. \text{ for } t \in [T_1, T] \tag{13}$$

$$\hat{P}_{t|t+1}^{\text{RE}} = \frac{bc}{(1-c)^2}\mu^x + \frac{b}{1-c}X_t \quad \text{for all } t \in [(T_1 - 1), (T-1)] \tag{14}$$

where the process characterizing X_t given by (3) and T_1 occurs at some time prior to T. By indicating that they are relevant during a time interval prior to T, the formulation in (13) and (14) provides a simple way to formalize the idea that any determinate model can at best approximate the price process and the market forecast during some stretch of time.

In order to represent the growth of knowledge, we suppose that at time T, representations in (13) and (14) cease to be adequate. There could then be a stretch of time, say, $[T, (T_2 - 1)]$, during which there is so much change in the process driving aggregate outcomes that these outcomes and the market's forecast cannot be adequately approximated in terms of some set of causal factors. Accordingly, we write the price process during this period of transition as:

$$P_t = \epsilon^p \quad \text{for all } t \in [T, (T_2 - 1)] \tag{15}$$

[19]The tests of the model's adequacy as an approximation of the way actual outcomes unfold over time are necessarily based on some finite samples. Thus, what is usually meant by the adequacy of the time-invariant REH model in (7) and (8) for the period $[T_1, T]$ is that, according to widely accepted statistical procedures, its structure has not been found to be inconsistent with time-series evidence.

[20]In general, such a condition could also apply to other outcomes, such as the market's forecast. We make such use of (12) later in this section.

where, ϵ^p is a white noise process. However, we suppose that such a transition period is eventually followed by a stretch of time, say $[T_2, T_3 - 1)]$, during which the new price process and the market forecast can be adequately approximated with the semi-reduced form:[21]

$$P_t = aV_t + f\hat{P}_{t|t+1} \quad \text{for all } t \in [T_2, (T_3 - 1)] \tag{16}$$

where, (a, f) are parameters, and V_t is the set of causal factors, among which at least some are different than those comprising in (3). However, for sinmplicity we also characterize them with a random walk with the drift, μ^v, and the disturbance term ε_t^v.[22]

We now show that representing participants' forecasting with REH after T presumes that they are irrational. To this end, we suppose that $\hat{P}_{t|t+1}^{\text{RE}}$ does adequately *approximate* the market's forecast during $[T, (T_3 - 1)]$.Using (15) as a characterization of the price process during the transition subperiod yields the following representation for the forecast error implied by the REH representation in (14):

$$FE_{t+1}^{\text{RE}} = P_{t+1} - \hat{P}_{t|t+1}^{\text{RE}} = -\alpha^{\text{RE}} - \beta^{\text{RE}} X_t + \varepsilon_{t+1}^p \quad \text{for all } t \in [T, (T_2 - 1)] \tag{17}$$

where, $\alpha^{\text{RE}} = \frac{bc}{(1-c)^2}\mu^x$ and $\beta^{RE} = \frac{b}{1-c}$. In order to obtain the sem-reduced form that characterizes the price process after the transition period, we substitute (14) into (16):

$$P_t = aV_t + f\left(\alpha^{\text{RE}} + \beta^{\text{RE}} X_t\right) \quad \text{for all } t \in [T_2, (T_3 - 1)] \tag{18}$$

Thus, the following expression represents the forecast errors during this period that are implied by the REH representation in (14); ,

$$FE_{t+1}^{\text{RE}} = \mu^{FE} + aV_t + \beta^{\text{RE}}(f-1)X_t + \varepsilon_t^{FE} \quad \text{for all } t \in [T_2, (T_3 - 1)] \tag{19}$$

[21] As we discuss in section 5, if the absence of any discernible regularity in time-series data were to continue indefinitely, no formal theory of co-movements between outcomes and causal factors that is compatible with rational decision-making would be possible.

[22] In Frydman and Goldberg (2007, chapter 15), we provide evidence that the growth of knowledge, as defined here, characterizes the processes underpinning the movements of major exchange rates. We show that there are stretches of time during which these movements can be approximately related to sets of causal factors drawn from a large class of monetary models. These stretches of time are punctuated by structural breaks. According to standard statistical criteria, for about a year after the break, there appears to be no relationship between the currency movements and the pre-break set of causal factors. Moreover, indicating the empirical relevance of the growth of knowledge, post-break relationships involve a different set of causal factors than the pre-change relationships.

where $\mu^{FE} = a\mu^v + (f-1)\alpha^{\text{RE}} - f\beta^{RE}\mu^x$ and $\varepsilon_t^{FE_t} = a\varepsilon_t^v + f\beta^{\text{RE}}\varepsilon_t^x$.Expressions in (17) and (19) imply that the forecast errors of REH representation during $[T, (T_3 - 1)]$ are biased on the average, they are correlated over time, and depend on X_t.

In the context of the REH model considered here, forecast errors and the variables in X_t represent participants' information set. Thus, were the REH representation to remain adequate after T, it would represent decision-making by irrational individuals who ignore systematic underperformance of their forecasts, thereby foregoing profit opportunities. This shows that REH models are abstractions of rational decision-making only in markets in which the knowledge underpinning the price process does not grow.

5 The Contingent Expectations Hypothesis

Lucas and Sargent persuasively argued that explaining regularities in time-series data and analyzing the consequences of changes in government policy require that macroeconomic models be compatible with rational decision-making. And yet it is self-evident that what their determinate REH models exclude – the growth of human knowledge – has been the central driver of our historical development, including economic outcomes. In view of the irrationality implied by reliance on REH in real-world markets in which knowledge clearly grows, we need to develop an REH analogue for such markets. In the next two sections, we formulate such an analogue, which we call the contingent expectations hypothesis (CEH), and use it to construct macroeconomic models that are compatible with rational decision-making and its consequences for aggregate outcomes in these markets.

Capitalist economies' hallmark is the establishment of powerful incentives that motivate individuals to search for new understandings of the world. In pursuit of profitable opportunities or other forms of personal advancement, individuals intermittently uncover new ways to deploy resources and forecast the consequences of their decisions. Indeed, such activities inherently require them to revise their understanding of the economy at times and in ways that they themselves, let alone economists, cannot fully anticipate in advance.

The growth of knowledge engendered by profit-seeking could involve relatively small modifications of existing production or marketing processes, or large changes, such as the commercial development of major technological innovations (for example, personal computers, the Internet, or new sources of energy). Change in the way the economy works may also be triggered by changes in the social context within which individuals make decisions, such as policy changes following presidential or parliamentary elections, or appointments of important officials, such as central bank governors. Changes in the social context also include major institutional and political developments, such as the establishment of GATT and other Bretton Woods institutions, the introduction of the euro, or German reunification.

Many of these developments can be seen as the cumulative outcome of many relatively small changes that do not alter in a discernible way the process underpinning outcomes. Thus, a determinate model may provide an adequate approximation of the economy over some stretch of time. However, sooner or later, the growth of knowledge or some other unanticipated developments would render any determinate model inadequate. This argument underpins one of CEH's two conceptual pillars.

> *Principle of Contingent Knowledge*
>
> For an economic model to be relevant for representing how a profit-seeking individual understands the process driving market outcomes and uses this knowledge to forecast these outcomes, it should recognize that this process is contingent: it is subject to change at times and in ways that cannot be fully anticipated.

Thus, in view of our arguments in the preceding section, for an economic model to be compatible with rational decision-making in real-world markets, in which knowledge grows, it must be partly open. But not all models in this class are compatible with individual rationality, because partly open models imply regularities in time-series data. Representing market participants' understanding of the process driving outcomes in ways that are incompatible with these regularities is tantamount to presuming that they forego profit opportunities. This reasoning underpins the second pillar of CEH.

> *Principle of Internal Coherence*
>
> A model's representation of market participants' understanding of the process driving aggregate outcomes cannot imply regularities in time-series data that conflict with the regularities implied by a model's account of those outcomes.

In the next section, we show how, similar to the requirement of internal consistency in REH models, this principle connects a partly open model's representation of forecasting to the specifications of its other components, and how this connection restricts changes in a model's structure over time.

6 Representing Rational Forecasting in Real World Markets

In Frydman and Goldberg (2007), we proposed imperfect knowledge economics (IKE) as an approach to building models that are partly open. IKE recognizes that no one can fully foresee that knowledge grows and thus no one can fully foresee how and when the process underpinning the market's forecast and aggregate outcomes might change. It does so by imposing qualitative and contingent constraints on how a model's structure might change from one point in time to another. Such constraints leave partly open the exact structures that might be needed to represent forecasting and market outcomes at every point in time. Nonetheless, they place sufficient discipline on the analysis to generate implications for time-series data.

6.1 IKE's Partly Open Models

We now use the simple example from section 3 to show how IKE opens economic models partially and how the principle of internal coherence is used in building IKE models that could be compatible with individual rational decision-making. For convenience, we recall the basic equations of our example:

$$P_t = b_t X_t + c_t \hat{P}_{t|t+1} \tag{20}$$

$$\hat{P}_{t|t+1} = \beta_t Z_t \tag{21}$$

where $\hat{P}_{t|t+1}$, X_t, and Z_t are defined as before, but the vector of parameters (b_t, c_t, β_t) not constrained to remain unchanged over time.

As time passes, economic policy and other aspects of the social context change in ways that influence how market outcomes unfold over time. Participants also alter how they understand the market process and how they forecast its outcomes, in part because of changes in the social context. The model represents such change with changes in its structure, that is, with changes in the parameters (b_t, c_t, β_t) and sets of causal variables (X_t, Z_t) in equations (20) and (21), as well as in the processes governing movements in these variables.

As it stands the model in (20) and (21) is completely open. Unless some constraints are imposed on how its structure might change over time, the model has no testable implications: it is compatible with any co-movements among the price and the causal variables. As such, the model cannot serve as the basis for representing rational forecasting.

IKE stakes out an intermediate position between the completely open and determinate models. Although its models are partly open to unanticipated change in the process underpinning market outcomes, IKE explores the possibility that individual decision-making does exhibit regularities. However, these regularities are context-dependent, qualitative, and contingent; that is, they begin and cease to manifest themselves at moments that no one can fully foresee. Thus, by design, all IKE models are consistent with the CEH's principle of contingent knowledge.

IKE's partly open models do not imply that outcomes and causal variables co-move in a fixed way. Nonetheless, they imply that there are stretches of time during which these variables' co-movements can be approximated by distinct qualitative relationships. By supposing that these stretches of time last long enough to be discernible by statistical or other methods, the principle of internal coherence selects the subclass of IKE models that represents rational forecasting in ways that are compatible with the hypothesized qualitative and contingent regularities.

6.2 Partly Open Representations of Rational Foreacsting

In order to illustrate how CEH represents market participants' forecasting strategies as partly open to unanticipated structural change, we consider a model of the process underpinning an economy's overall inflation rate. Our example, which is based on a New Keynesian specification, is stylized and simplified. But it shows the qualitative and contingent constraints of an IKE model, and how the principle of internal coherence is used to restrict both a model's structure and the change in this structure over time.

We suppose that the inflation process can be represented along the lines of Calvo (1983) and Rotemberg (1985):

$$P_t = b_{1t} Y_t + b_{2t} V_t + c_t \hat{P}_{t|t+1} \tag{22}$$

where P_t denotes the inflation rate between $t-1$ and t, Y_t is overall output, V_t represents marginal cost considerations other than those that are captured by Y_t, $\hat{P}_{t|t+1}$ is given by (21), and the parameters b_{1t}, b_{2t}, and c_t are all positive. We assume that V_t is related to the need for firms to obtain credit to finance their wage bills. Labor costs, and thus V_t, depend on the cost of credit, which we assume is determined by the real interest rate, R_t. A rise in the real rate, therefore, has an inflationary impact on aggregate supply by increasing the marginal cost of labor.[23]

[23] See Van Wijnbegen (1983, 1985) and Neumeyer and Perri (2005) for models in which credit financing of working capital plays a key role in business cycle fluctuations.

We specify output with a simple IS curve that relates Y_t to the real interest rate. This specification enables us to express the inflation process as follows:

$$P_t = b_t R_t + c_t \hat{P}_{t|t+1} \tag{23}$$

where the algebraic sign of b_t would be positive if the inflationary supply-side effect of a change in R_t was greater than the deflationary demand-side effect, and negative if the converse were true.

In the absence of any restrictions on how the model's structure could change over time, the model has no implications for time-series data. In formulating such restrictions in an IKE model, the economist looks for empirically relevant regularities in both how the underlying (semi-reduced-form) market process and participants' understanding of this process change, as represented by shifts in b_t and c_t on the one hand and changes in β_t and the composition of Z_t on the other, respectively. The principle of internal coherence implies a connection between the model's representations of change on these two levels.

6.2.1 Change in the Underlying Economy

Consider, first, the modeling of structural change in the underlying economy. We suppose that there are stretches of time of irregular length during which little or no discernable change occurs, that is, during these stretches the underlying economy can be approximated with constant parameters, b^i and c^i, where $i = 1, 2$...ith sub-period's linear approximation.[24] Shifts between these sub-periods or linear pieces of the model represent the impact on the underlying economy of new economic policies or other changes in the social context.

Empirical observation suggests that such a piece-wise linear specification accords well with how developments in the underlying economy unfold. For example, in the context of our inflation example, firms' nominal interest payments are deductible as expenses under current law, implying that the real cost of working capital depends on their marginal tax rates. A lower marginal tax rate would lower the size of these deductions, thereby raising the cost of working capital. Shifts in tax policy, therefore, would influence the relative strength of the supply-side and demand-side effects of changes in the real interest rate on inflation.

[24] As we discussed in section 4, as long as changes are not discernable by statistical or other methods, the linear approximation will be adequate. But sooner or later changes will cumulate to become discernable as a structural break. This structural breaks would generally be followed by sub-periods of transition during which no linear approximation would be adequate. In the formulation here, we abstract from such transitions.

Shifts in tax policy occur infrequently, so we would expect rather long stretches of time in which a linear approximation would characterize the underlying economy. But, when shifts in tax policy do occur, we would need to allow for structural change in our representation in (23) in order to account for how interest-rate movements impacted inflation. In fact, if the change in tax policy were large enough, it could be associated with a shift in the sign of the correlation between inflation and the real rate, that is, a switch in the sign of b_t. Changes in tax policy and other important features of the social context may also influence the sensitivity of the inflation rate to changes in the market's forecast; that is, we may need a different $c_t > 0$ to represent the inflation process. No one can fully foresee when and how such change will occur, let alone its impact on the inflation process.

Shifts in tax policy occur infrequently, so we would expect rather long strecthes of time in which a stable b_t would characterize well the underlying economy. But, when shifts in tax policy do occur, we would need to allow for stuctural change in our representation in (23), that is, for new linear pieces to account for how interest rate movements impacted inflation. In fact, if the change in tax policy was large enough, it could be associated with a shift in the sign of the correlation between inflation and the real rate, that is, a switch in the sign of b_t.[25] Changes in tax policy and other important features of the social context may also influence the sensitivity of the inflation rate to changes in the market's forecast, that is, we may need a different $c_t > 0$ to represent the inflation process. No one can fully foresee when and how such change will occur, let alone the impact of this change on the inflation process.[26]

Our IKE model recognizes that this unpredictability by leaving open the exact timing and nature of structural change. But, in order to derive implications for time-series data from our IKE model, it must suppose that the sub-periods in which the linear approximations of the underlying economy tend to last long enough for relationships to be discernable in the data. The extent to which they do depends on whether market participants' forecasting behavior is also assumed to display some regularity.

[25] For example, the Tax Reform Act of 1986 eliminated the investment tax credit, dramatically changed depreciation allowances, and lowered the top federal statutory rate of corporate income tax from 46% to 34% (which was increased in 1993 to its current rate of 35%). Cohen et al. (1999) discuss the impact of such tax policies on the user cost of capital and the costs of inflation.

[26] Paul Volcker's decision in 1979 to target monetary aggregates instead of interest rates, and Germany's decision, a decade later, to reunify, are other examples of shifts in the social context that may require new structures to represent their impact on the underlying economy.

6.2.2 Market Participants' Forecasting Strategies

The piece-wise linear model of the inflation process in (23) implies that, in general, fluctuations in the inflation rate in any sub-period depend in part on fluctuations in the real interest rate. Relying on Muth's insight that an economist's model can be used to represent the market's understanding, we include R_t as one of the causal variables in our representation of the market's forecast in (21). If we consider a stretch of time in which we suppose that the demand-side effect from changes in the real interest rate dominates the supply-side effect, by setting $b^i < 0$, then we embody this understanding to our representation of the market's forecast; that is, we set the weight on this variable, which we denote by β_t^r, to be positive, regardless of how it varies over the sub-period. Conversely, if we suppose that $b^i > 0$, we set $\beta_t^r > 0$.

Iterating equation (23) forward one period shows that the current inflation rate depends on the one-period-ahead real interest rate. If we were to impose REH, we would carry out this iteration to time infinity. As with our earlier example in section 3, we would specify change in the semi-reduced form structure of the model as determinate, assume a determinate process for R_t (for example, a random walk), and then solve for P_t in terms of R_t. This would yield a representation for the market's forecast analogous to the one in (8).

By contrast, an IKE model accords expectations a partly autonomous role in driving outcomes. Doing so recognizes that, in forecasting the real interest rate and inflation, market participants may rely on news about a range of informational variables – for example, overall income or credit, economic and political developments, the conduct of monetary policy, or the results of state and federal elections – as well as psychological considerations such as confidence and optimism.[27] Consequently, we would expect that, in order to represent the market's forecast of inflation, we would need to include other casual variables in our specification of Z_t.

To some extent, the inclusion of other variables in our representation would be guided by theoretical considerations. We have kept our example simple by assuming a one-equation model of inflation. But a more complete model would recognize that inflation is governed by a broader macroeconomic process involving other endogenous variables, such as income and interest rates. The causal variables that entered the other equations of the

[27] There is much evidence that in forecasting outcomes in asset markets, participants rely on a broad range of fundamental, psychological, and technical considerations. See, for example, Mangee (2013) and Sullivan (2013), which construct novel datasets based on scoring daily market wrap stories from *Bloomberg News* and *The Wall Street Journal*, respectively.

model would be part of our understanding of the economy, and we would include them in our representation of the market's forecast. The autonomous role for expectations also enables an economist to base his representation of participants' decision-making on empirical observation. For example, Frydman *et al.* (2013a,b) make use of information contained in market reports from *Bloomberg News* and *The Wall Street Journal* in specifying a piece-wise linear model of stock and currency markets (see footnote (see footnote **??**).

6.2.3 Revisions of Forecasting Strategies

The representation of the market's forecast in (21) implies that there are two key factors that underpin its unfolding over time: revisions of participants' forecasting strategies – changes in β_t, which could include changes in the composition of the set of causal variables Z_t – and movements in the informational variables:

$$\Delta \hat{P}_{t|t+1} = \Delta \beta_t Z_t + \beta_{t-1} \Delta Z_t \qquad (24)$$

where Δ denotes a first difference.

In general, market participants decide to revise their forecasting strategies for various reasons, including the performance of their current strategies and news about a whole host of macroeconomic, political, and other developments that can lead them to rethink how the causal variables affect inflation. This decision also depends on psychological factors, such as the confidence that they have in their current strategies and their intuition regarding the possibility that structural change in the underlying economy has occurred or may occur over the forecast horizon. And, of course, there is a great diversity in how market participants might revise their forecasting strategies.

It would seem, therefore, that revisions of how the market's forecast is related to a set of casual variables are unlikely to display any regularity that can be formalized with a mathematical model. However, the principle of internal coherence implies that they do, given our piece-wise linear representation of the underlying economy.

6.2.4 Internal Coherence as a Modeling Device

In an REH model, the connection between the representations of the underlying economy and the market's forecast is very tight; given specifications of the non-expectational components of the model, REH selects a representation of the market's forecasting strategy whose parameters and variables are determined exactly by cross-equation restrictions. If the model allows for any structural change in the underlying economy, internal consistency implies exactly how the parameters and/or composition of variables that enter the model's specification of forecasting will change. Expectations play no autonomous role in driving outcomes in the model, so there is no room to incorporate empirical observations of how participants in real-world markets actually alter their forecasting strategies.

In an IKE model, by contrast, the principle of internal coherence implies a less stringent connection between the model's representations of the underlying economy and the market's forecast. This is because its qualitative and contingent constraints on structural change do not select one structure at every point in time to represent change in the market process.

To see how this principle is used as a modeling device, we take first differences of (20) and substitute in (24), yielding the following expression for how inflation unfolds over time:

$$\Delta P_t = b^i \Delta R_t + c^i \left(\Delta \beta_t Z_t + \beta_{t-1} \Delta Z_t \right) \tag{25}$$

where $i = 1, 2....$ Internal coherence in this IKE model imposes restrictions on $\Delta \beta_t$ so that the qualitative and contingent regularities that are implied by this reduced-form equation are consistent with the model's representation of the understanding of the inflation process that underpins the market's forecast.

Consider a sub-period in which the demand-side effect from changes in the real interest rate is assumed to dominate the supply-side effect; that is, we set $b^i < 0$. Imputing this understanding to our representation of the market's forecast, we restrict $\beta_t^r < 0$; that is, we portray the market's understanding of the inflation process over the sub-period to involve a qualitative relationship in which P_t and R_t co-move negatively. The implications of our model for the relationship between inflation and the real interest rate can be expressed as follows:

$$\Delta^R P_t = \left(b^i + c^i \beta_{t-1}^r \right) \Delta R_t + \Delta \beta_t^r R_t \tag{26}$$

where Δ^R denotes a change in P_t that results solely from a change in R_t. We define the status-quo impact on inflation to stem only from changes in the real interest rate, ΔR_t, with no change in structure—that is, when $\Delta \beta_t^r = 0$.. Movements in R_t can also affect inflation by leading market participants to alter their forecasting strategies. Depending on how we restrict these revisions, the implications of our reduced-form equation may or may not conflict with the market's understanding that P_t and R_t co-move negatively over the sub-period.

Internal Incoherence Internal incoherence would result if we assumed that movements in real interest rates led market participants to revise their forecasting strategies in ways that implied an impact on inflation that impeded the status-quo impact, that is,

$$\Delta\beta_t^r = \gamma_t \frac{\Delta R_t}{R_t} \text{ for subperiod } i \tag{27}$$

where γ_t is positive and larger in magnitude than $\left(b^i + c^i\beta_{t-1}^r\right)$. With such non-reinforcing revisions, the model would imply a qualitative relationship between P_t and R_t that involved positive co-variation, while its representation of the market's understanding would imply the opposite.

Internal Coherence To ensure internal coherence in our IKE model in terms of the relationship between inflation and the real interest rate, we need one of two constraints. One supposes that changes in the real rate lead market participants to revise their forecasting strategies in ways that tend to reinforce the status-quo impact over the sub-period. For example, this would be the case if we set $\gamma_t < 0$ in (27). The other restriction supposes that revisions tend to be what we call "guardedly moderate," in the sense that,

$$|\Delta\beta_t^r| < \beta_{t-1}^r \frac{|\Delta R_t|}{R_t} \text{ for subperiod } i \tag{28}$$

where $|\cdot|$ denotes an absolute value. This latter constraint on structural change is consistent with both reinforcing and non-reinforcing revisions. Assuming that it largely holds over the sub-period supposes that market participants are reluctant to alter their forecasting strategies in ways that would have the potential to offset completely the status-quo impact and thus their understanding that P_t and R_t co-move negatively.

The qualitative constraints in (27) and (28) are consistent with myriad possible revisions of forecasting strategies between adjacent points in time – and thus with myriad possible new structures at t. But each of these new structures, conditional on β_{t-1}^r, implies that inflation and the real interest rate move in opposite directions. Consequently, if the constraint in either (27) or (28) was assumed to be largely satisfied over the sub-period, the reduced-form equation in (25) would imply that inflation tended to co-move negatively with the real interest rate, and the model would be internally coherent.

Internal coherence also constrains how other causal variables can be included in the representation of the market's forecast. Because these variables do not enter the semi-reduced-form equation, the model provides no guide to how they might impact inflation. We must rely on theoretical or empirical considerations outside of the model. For example, such considerations may indicate that movements in the unemployment rate often lead participants to revise their inflation forecasts in the opposite direction. Suppose that, by setting the weight on this variable, β_t^u, to be negative, we embodied this understanding in our representation of the market's forecast during the sub-period. The resulting IKE model would be internally coherent if we restricted $\Delta\beta_t^u$ so that these revisions largely satisfied the qualitative constraints in (27) or (28) over the sub-period.

6.2.5 Qualitative and Contingent Regularities

Internal coherence in the model does not require that revisions satisfy the constraints in (27) and (28) at every point in time. Indeed, the model does not specify exactly when they do. What is required is that one of these constraints is satisfied frequently enough for the model to imply qualitative regularities in how inflation co-moves with the causal variables, and that these regularities are consistent with the understanding that the model uses to represent the market's forecast.

In fact, the contingent nature of the constraints in (27) and (28) is required for ensuring internal coherence, given the model's contingent piecewise linear specification for the underlying economy. To see this, we contemplate an unforeseen change in tax policy that we assume is large enough to cause a shift in the underlying economy. We also suppose that a new period during which another linear approximation becomes adequate and in which the supply-side effect from changes in the real interest rate is larger than the demand-side effect – that is, in this new sub-period, $b^i > 0$. If we assume that market participants' understanding of the inflation process shifts roughly at the same time, we would need to assume a change in β_t^r at the break point that is positive and large enough to ensure that the algebraic sign of β_t^r switched from negative to positive. In general, such a large revision may conflict with the guardedly moderate constraint in (27). Depending on the sign of ΔR_t, it may also conflict with the reinforcing constraint in (28).[28]

In contrast to most REH models, our CEH-based IKE model does not produce a sharp prediction about inflation. But it does constrain structural change sufficiently to generate implications for time-series data. These implications are qualitative and contingent on developments in the underlying

[28] See Edwards (1968). For an application in a determinate behavioral finance model, see Barberis et al (1998).

economy and on how market participants' understanding of these developments unfolds. The model implies that there are distinct sub-periods in the data that are approximated by linear relationships between inflation and a particular set of causal variables that includes the real interest rate. The model specifies neither the duration of these sub-periods nor how many of them may be present in historical data. But, it implies that if shifts in the underlying economy occur infrequently enough, statistical procedures should uncover structural change and sub-periods in the data characterized by particular qualitative relationships that may involve a reversal in the way inflation co-moves with the real rate.

7 Concluding Remarks

In this paper, we have proposed the contingent expectations hypothesis as way to build models that are compatible with rational decision-making in real world markets. In contrast to the usual dualism concerning the importance of fundamental and psychological considerations, which is largely an artifact of determinate models, partly open CEH-based models incorporate both factors into representations of rational decision-making. This conclusion suggests that some of the behavioral findings concerning how individuals actually behave in market settings are consistent with rationality, in the sense that individuals rely on their understanding of the economy in pursuing their objectives.

As the example of such a model in the preceding section shows, CEH's principle of internal coherence implies qualitative and contingent constraints on representations of forecasting strategies. One of these constraints is that, faced with the inherent contingency of their own knowledge, participants in real-world markets tend to revise their thinking about how fundamentals matter in "guardedly moderate" ways: there are stretches of time during which they either maintain their strategies or revise them gradually.

Such revisions do not generally alter, in substantial ways, the set of fundamentals that participants consider relevant and/or their interpretation of their influence on future outcomes. Psychological studies have uncovered evidence that individuals revise their beliefs in the face of new evidence gradually.[29] This behavioral regularity was also emphasized by Keynes (1936, p.152) in his analysis of financial markets: regardless of whether participants in these markets are bulls or bears, they tend to assume that the "existing state of affairs will continue indefinitely, except in so far as we have specific

[29] See Edwards (1968). For an application in a determinate behavioral finance model, see Barberis et al (1998).

reasons to expect a change." Even when a participant does "have specific reasons to expect a change," it is entirely unclear what new forecasting strategy, if any, she should adopt.[30]

Behavioral findings of REH models' empirical shortcomings have not led to the jettisoning of REH. Instead, behavioral economists and others have maintained this hypothesis as the way to represent rational forecasting. Consequently, they have interpreted their findings as a symptom of individuals' irrationality. The results of this paper point to the opposite conclusion: behavioral findings that are implied by the principle of internal coherence—for example, that individuals tend to revise their forecasting strategies gradually—are a symptom of rational forecasting in real-world markets.

REH models have profoundly shaped our thinking about markets, and have altered how we regulate them and conduct economic policy. In this paper, we have shown that these models are incompatible with rational decision-making in real-world markets. Remarkably, the arguments by Muth, Lucas, and Sargent that, in explaining regularities in time-series data, a macroeconomic model should not imply individuals' irrationality call into question the empirical and policy relevance of REH-based implications.

In order to ascertain whether any of the REH-based implications of determinate models can be related to rationality, we need to reexamine them in the context of partly open CEH-based models. Such a reexamination may also help us to resolve many of these models' empirical shortcomings.

In view of the epistemological and empirical difficulties of determinate macroeconomic models, we face a clear choice. We can continue to study "markets" in which knowledge does not grow and search for a determinate structure that will ultimately explain exactly how market outcomes unfold over time. Or we can place at the center of macroeconomic research partly open models that recognize that determinate structures are inherently out of reach for economic analysis, but that nonetheless provide a way to build macroeconomic theory that is compatible with rational decision-making and thus can explain regularities in time-series data.

Nearly a century ago, Knight (1921, p. 198) articulated clearly what this choice entails. As he put it, "if all changes were to take place in accordance with invariable and universally known laws, [so that] they could be foreseen for an indefinite period in advance of their occurrence,... profit or loss would

[30] See Frydman and Goldberg (2013b) for an extensive discussion of how empirical findings from the other social sciences and psychology can be used in building the microfoundations of macroeconomic models. Frydman and Goldberg (2013c) formalize this empirical insight and use it to represent revisions of forecasting strategies in a model of asset-price swings. In Frydman and Goldberg (2013d), they use the principle of internal coherence in building an IKE model of swings and risk that is compatible with rational decision-making.

not arise." For Knight, as for us, "it is our imperfect [and contingent] knowledge of the future, a consequence of change, not change as such, which is crucial to the understanding" how profit-seeking market participants make decisions and how aggregate outcomes unfold over time.

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