Aging, Output per capita and Secular Stagnation

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---- Very Preliminary ----

Abstract

This paper shows that aging has positive effect on output growth per capital at positive interest rates, due to capital deepening. This is consistent with cross country data. This correlation, however, reverses itself if the process goes to far (as in post 2008) and a negative real interest rate is needed to clear the market. In that case, the data shows that aging has negative effect on output growth per capita. This new cross-country correlation is predicted by the secular stagnation hypothesis suggested by Summers (2014). We review the cross-country correlation in the data and highlight the mechanisms in a stripped down two generation OLG model with capital and nominal frictions.

[1] Eggertsson: Brown University and NBER; Lancaster: Brown University, Summers: Harvard University. We thank INET for financial support and Acemoglu and Restrepo for generously sharing their data with us. We also thank Ian Tarr for research assistance.

1. Introduction

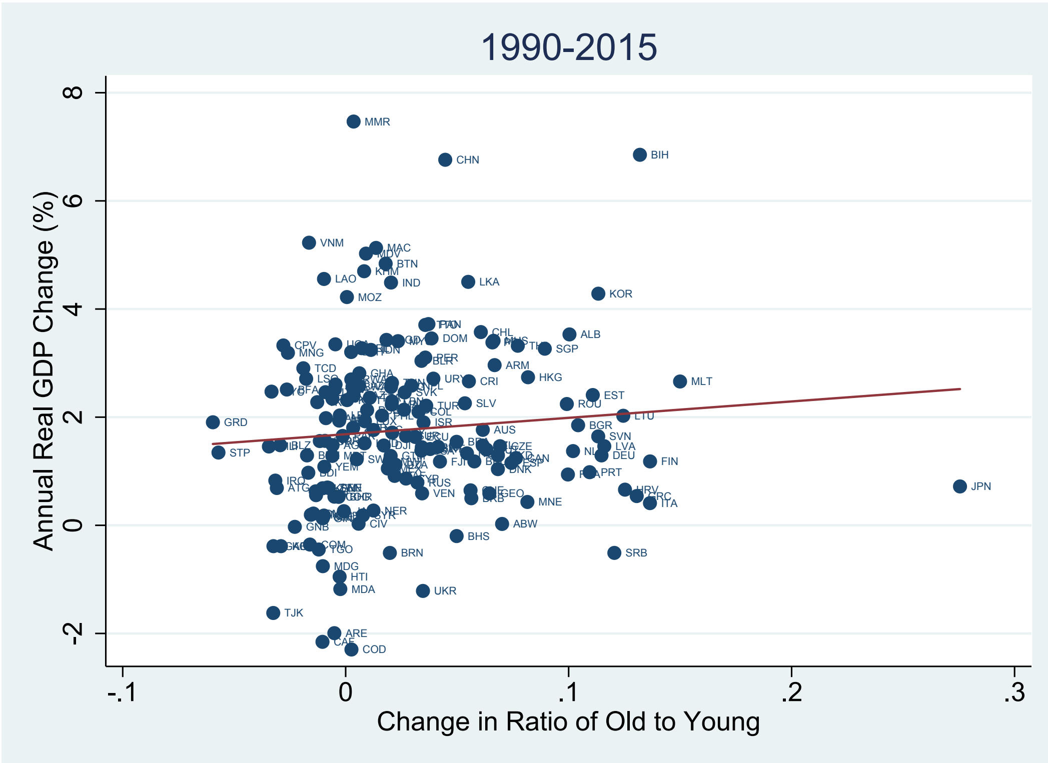

Figure 1 Aging and annual GDP growth in the period 1990-2015

In recent years there is an emerging literature that connects aging with slowdown in growth of GDP per capita.

At the most basic intuitive level, one might expect a negative correlation. As the population ages, then there are relatively fewer members of the population that are part of the workforce. Hence, holding other factors of production fixed, one should expect a decline in GDP per capita, thus translating into slower GDP growth per capita over a period of time.

Recently, however, Acemoglu and Restrepo (2017) report a somewhat surprising correlation that is replicated in Figure 1. On the x axis is the change in the ratio of old to young workers in the period 1990-2015. On the y axis, there is the average annual GDP growth in the period 1990-2015. We discuss the details underlying the figure in Section 3.

The slope of a basic regression line is positive in the figure. It appears that as societies age, then there is an increase in their average GDP per capita growth rate over the past quarter of a century. Acemoglu and Restrepo (2017) consider various alternative specification to a basic regression plotted above, and country subsamples but conclude that this appear to be a robust correlation. They interpret this as evidence contradicting the so-called secular stagnation hypothesis (see Summers (2013, 2014) or Eggertsson and Mehrotra (2014)). Here we argue the opposite.

The explanation Acemoglu and Restrepo (2017) offer is a particular type of production function that incorporates the arrival of labor-replacing technologies post 1990, most prominently identified with robotics and artificial intelligence. The argument is that with sufficiently abundant capital, a shortage of younger and middle-aged workers trigger greater adoption of new automation technologies so that the negative effect of labor scarcity is completely neutralized or even reversed[2]. Importantly, Acemoglu and Restrepo's model assumes a fixed real interest rate, as would for example be predicted by the standard neoclassical growth model.

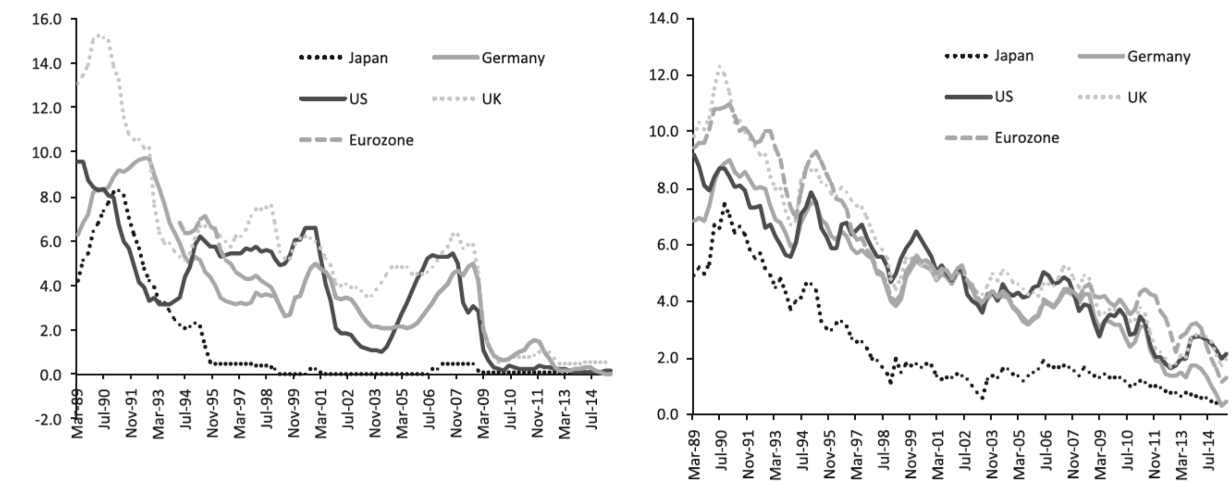

Figure 2: Short and long term interest rate since 1990

In this paper we make two points.

First, in order to explain a pattern such as in Figure 1 one does not need to resort to unconventional production functions that incorporate robots or artificial intelligence. Instead, all that is needed, is to introduce a simple OLG structure in which the interest rate fall as number of old people increase relative to young due to relative increase in savings. The decline in the real interest rate can in general equilibrium lead to a deepening of the capital stock so as to equate marginal product of capital to the equilibrium interest rate. In Figure 2 we see that the interest rates have been continuously declining since 1990 consistent with this explanation.

The second point, which is perhaps more interesting, relates to the connection of the data presented in Figure 1 to the secular stagnation hypothesis. The secular stagnation hypothesis as presented in Summers (2014) and Eggertsson and Mehrotra (2014) is *not* one that predicts that aging per see needs to lead to lower growth, as the preceding paragraphs highlights. What the secular stagnation hypothesis predicts, however, is that if there are forces (such as aging, but also a host of others such as debt deleveraging, increase in inequality, fall in the relative price of investment or a slowdown in productivity, see literature cited above) that are strong enough so that the real interest rate needed to clear the market is *negative, and if the central bank targets low inflation,* then the ZLB could be reached and those countries experiencing "excess" savings

[2] A similar argument is made by Cutler, Poterba, Sheiner and Summers, L. H., (1990).

will see lower growth. Figure 1 is not very instructive on this point, for it covers the period 1990-2015, while the zero bound did not become binding (aside from in Japan) until in 2008. What the secular stagnation hypothesis *does predict,* is that those countries that are aging faster in 2008, and are experiencing low inflation, would have larger excessive savings, on average, and thus presumably experience a deeper recession post 2008 if they hit the ZLB. Once we look closer at the data behind Figure 1 we will see that this hypothesis is in fact borne out and the positive correlation between aging and growth is therefore driven by the data prior to 2008.

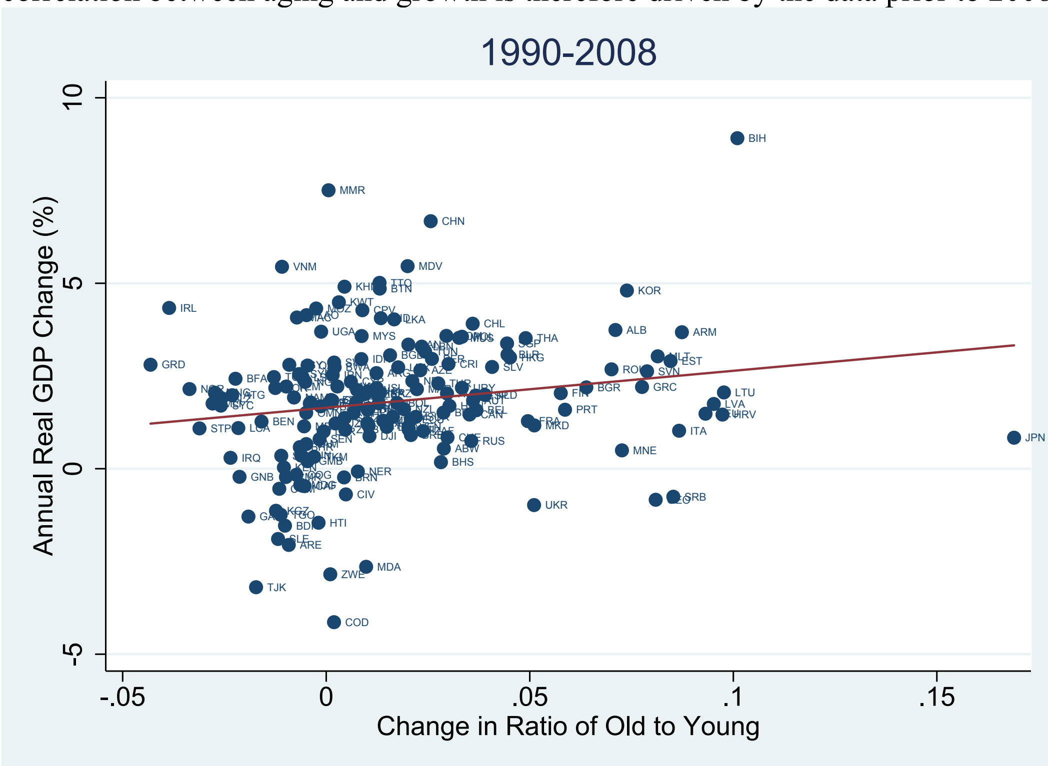

Figure 3 Growth and Aging pre ZLB episode

To elaborate on the second point, Figure 3 depicts the relationship between growth and aging from 1990-2008. As the figure suggest, there is still an upward sloping relationship between aging and growth, just as in Figure 1, but now it is even stronger than before. Perhaps even more interesting is to look at the period 2008-2015, shown in Figure 4, post crisis when several countries hit the ZLB. There we see that this correlation is reversed – as suggested by the secular stagnation hypothesis. Now the correlation between aging and growth is *negative* and this slope is estimated to be statistically significant. According to the secular stagnation hypothesis this relationship is predicted to follow from the fact that those countries with increasing ratio of old to young population would tend to have higher savings and investment imbalance, and thus experience a greater downturn at the ZLB than countries with a relatively younger workforce.

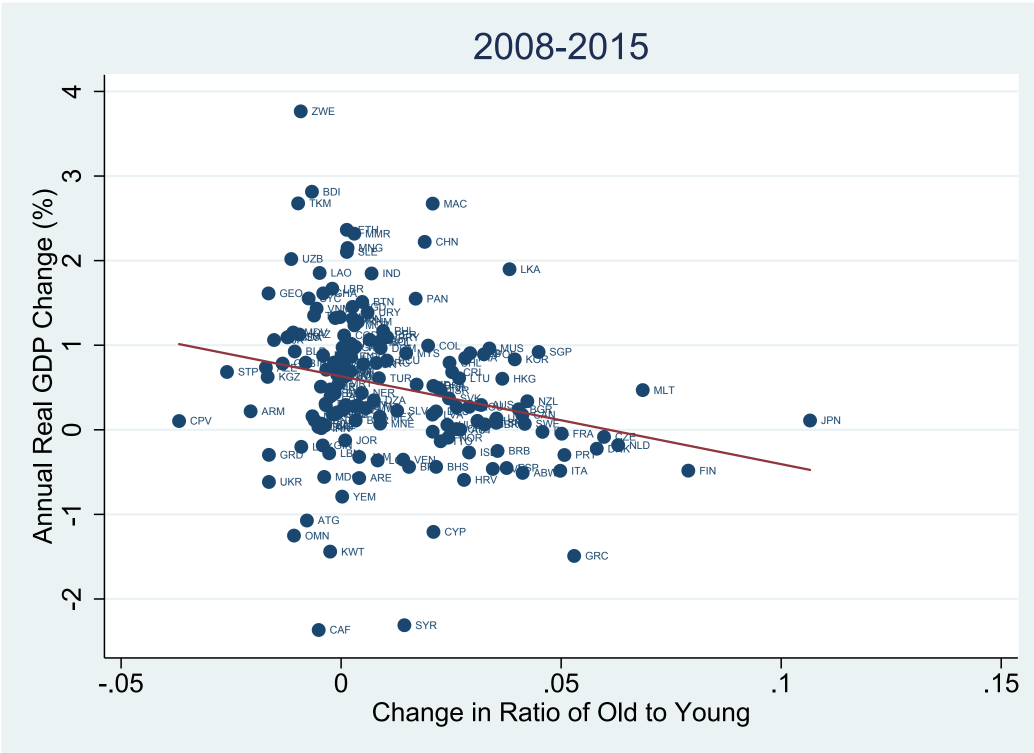

Figure 4 Relationship between growth and aging post crisis

The interpretation just offered in Figure 3 relies on the hypothesis that some of the countries in the sample were constrained by the ZLB, in which case the sign of the slope of the regression line is predicted to reverse itself. To further explore this idea Figure 5 separates out the countries in which the nominal interest rate was at or below 0.5 at some point during this period (and thus arguable constrained by the ZLB) while Figure 6 shows the remaining countries. Driving the statistically negative correlation in Figure 4 is in fact the countries at the ZLB, as shown in Figure 5, while the correlation is not statistically significant in the case of countries that where not at the ZLB (Figure 6)

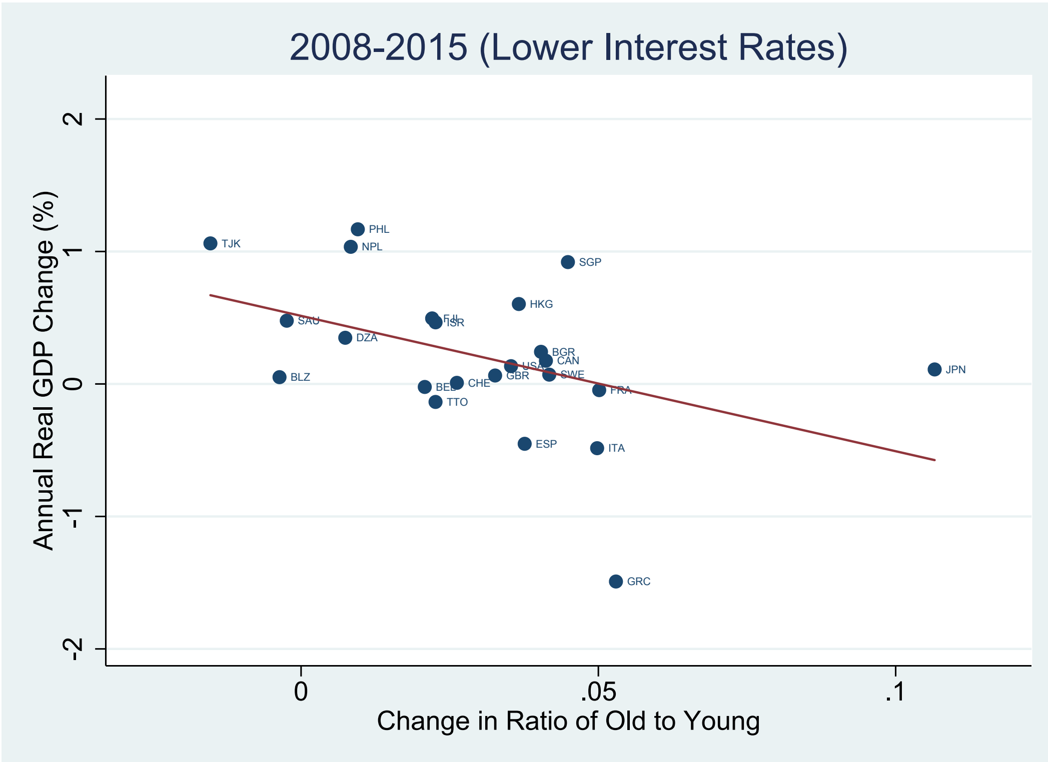

Figure 5 Aging and growth post crisis: Low interest rate countries

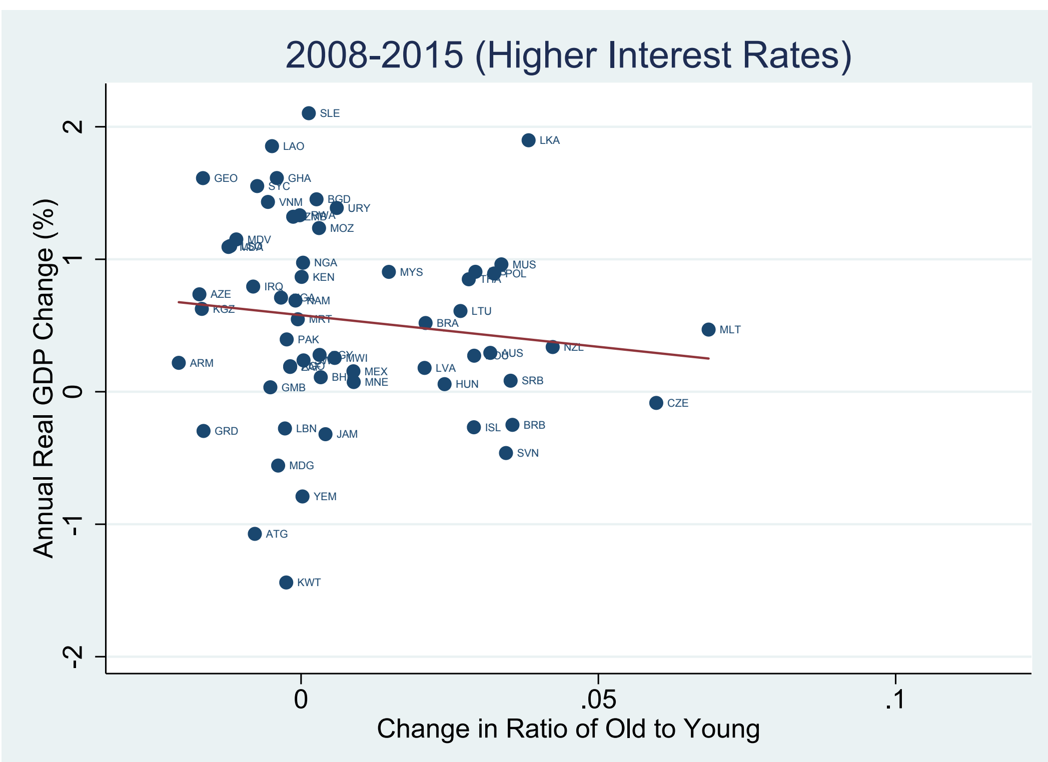

Figure 6 Aging and growth post crisis: High interest rate countries

Figure 7: Aging and Annual GDP Growth: High interest rate countries

The paper proceeds as follows. In section 2 we lay out a stripped down OLG model that formalizes the two theoretical points made above that relates the effect of aging on GDP per capita pre and post 2008. We strive for simplicity rather than generality, as we believe the forces at work will operate in a large class of models. In section 2 we report in more detail on the basic figures shown in the introduction, considering a variety of controls. Section 3 concludes.

2. A simple model

2.A 1990-2008: Capital Deepening

We first consider a simple model that can rationalize Figure 3, i.e., aging leads to a decline in real interest rates and capital deepening that is strong enough to explain an increase in GDP per capita. Consider an overlapping generation model with two generations, young and old. The young earn labor income, the old do not. The young can invest in capital and sell in old age for retirement.

A generation born at time t is of size N_t^y and has the utility function

(1)

$$U_t = \frac{1}{1-\sigma}(C_t^y)^{1-\sigma} + \beta\frac{1}{1-\sigma}(C_{t+1}^o)^{1-\sigma}$$

and faces the budget constraint when young.

(2)

$$C_t^y = w_t\bar{l} \quad - k_{t+1} - \tau_t$$

where w_t is the real wage rate, $\bar{l}$ is a fixed labor endowment and k_{t+1} is the capital saving of the young that can be used for production in the next period and τ_t is taxes. The budget constraint of the old is

(3)

$$C_{t+1}^o = R_{t+1}^k k_{t+1}$$

where R_{t+1}^k is the gross return on capital. For simplicity we assume capital fully depreciates, even if this is not essential. We further assume $\sigma = 1$, specializing in log utility, but will comment on how different values of σ matter. The young satisfy a consumption Euler Equation given by

(4)

$$\frac{1}{C_t^y} = \beta E_t \frac{R_{t+1}^k}{C_{t+1}^o}$$

while the old consume all their income. We assume that the growth of the population is

$$N_{t+1}^y = (1 + g_t)N_t^y$$

Let us define the aging parameter as the ratio of old versus young at time t, i.e.

$$A_t = \frac{N_{t+1}^o}{N_{t+1}^y} = \frac{N_t^y}{N_{t+1}^y} = \frac{1}{1+g_t}$$

With a little bit of algebra, the aggregate saving is
(5)

$$K_{t+1}^s = N_t^y k_{t+1}^s = \frac{\beta}{1+\beta}(w_t N_t^y \bar{l} - N_t^y \tau_t)$$

We assume perfectly competitive firms, that have a constant return Cobb-Douglas function $Y_t = K_t^\alpha L_t^{1-\alpha}$, thus satisfying the first order conditions

$$R_t^k = \alpha \frac{Y_t}{K_t}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_t}$$

Combining these conditions and using the production function with some manipulations, let us define aggregate demand for capital *per worker at time t+1* as[3]

$$\frac{K_{t+1}^d}{N_{t+1}^y} = (\frac{\alpha}{R_{t+1}^k})^{\frac{1}{1-\alpha}}$$

Using (5), and assuming that taxes are proportional to steady state labor income,[4] we can similarly define aggregate supply of capital per worker at time t+1 as

$$\frac{K_{t+1}^s}{N_{t+1}^y} = \frac{N_t^y}{N_{t+1}^y} k_{t+1}^s = \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(\frac{\alpha}{R_t^k})^{\frac{\alpha}{1-\alpha}} A_t$$

An equilibrium is now defined as when the demand and supply of capital are equated. As the model is of exponential form, it is linear in logs, and can be solved in closed form. Define $\tilde{k}_{t+1}^d \equiv log\frac{K_{t+1}^d}{N_{t+1}^y}$ and $\tilde{R}_{t+1}^k \equiv logR_{t+1}^k$ etc. As the dynamics are not fundamental to our point,[5] we can write the demand and supply for capital in steady state as

$$\tilde{k}^d = \frac{1}{1-\alpha}\log\alpha - \frac{1}{1-\alpha}\tilde{R}^k$$

and

$$\tilde{k}^s = \log\frac{\beta}{1+\beta}(1-\tau)(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} - \frac{\alpha}{1-\alpha}\tilde{R}^k + \tilde{A}$$

[3] See Appendix for step by step derivation.
[4] We assume that $\tau_t = w \quad \bar{l}\tau$
[5] See Appendix for derivation of full dynamic system.

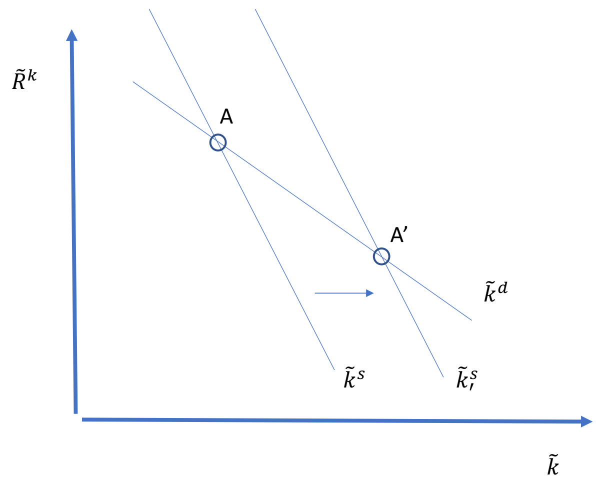

Figure 8 Aging and capital per worker

that are plotted up in Figure 1. The interpretation of the demand for savings is straight forward. The demand for capital is higher the lower is $\bar{R}^k$ as shown in the figure, as capital becomes relatively cheaper. The aggregate supply of capital is also downward sloping, but with a steeper slope. The reason is that the young are earning more income, with the higher capital stock in the steady state, and thus supplying more savings in equilibrium. The strength of this effect does depend on $\sigma = 1$ (which determines the relative strength of the income and substitution effect). With high enough σ then the supply curve for capital can be upward sloping, and thus the effect is not as strong. In either case, an increase in aging will shift out the supply of capital, and thus moving along the aggregate demand curve, increasing the demand for capital at lower real interest rate, moving from point A to point A'.

The steady state interest rate is given by[6]

$$\tilde{R}^k = \log \frac{\frac{\alpha}{1-\alpha}}{\frac{\beta}{1+\beta}(1-\tau)} - \tilde{A}$$

and similarly, we can solve for capital to yield

$$\tilde{k} = \frac{1}{1-\alpha}\tilde{A} + \frac{1}{1-\alpha}\log[\frac{\beta}{1+\beta}(1-\tau)(1-\alpha)]$$

Accordingly, we see, that unlike in the standard representative agent model, aging instead leads to higher capital labor ratio, a capital deepening, and a reduction in the real interest rate.

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[6] The convergenge to this new steady state is given by the dynamic equation $\tilde{R}^k_{t+1} = \alpha\, \tilde{R}^k_t - (1-\alpha)\tilde{A}_t + (1-\alpha)\log \frac{\frac{\alpha}{1-\alpha}}{\frac{\beta}{1+\beta}(1-\tau)}$

The strength of this effect on the capital labor ratio will depend on the capital share in the economy, and more generally on σ. Unambiguously, however, aging will lead to an increase in output per worker.
What about output *per capita*? Now there are two offsetting forces at play. On the one hand output *per worker* increases. On the other hand, there are *labor force per capita* decreases. Denoting output per capital by y^{pc} and its log with $\tilde{y}^{pc}$, we can express the difference between two steady states (denoting the second by ') as

$$\tilde{y}'^{pc} - \tilde{y}^{pc} = \alpha(\tilde{k}' - \tilde{k}) - \log\frac{1+A'}{1+A}$$

where the first term is positive and reflect higher capital per worker in the new steady state, while the second term reflects the reduction in labor input due to aging, which is negative. Substituting our solution derived from k we obtain[7]
(6)

$$\tilde{y}'^{pc} - \tilde{y}^{pc} = \frac{\alpha}{1-\alpha}\left(\log\frac{A'}{A}\right) - \log\frac{1+A'}{1+A} \approx \left(\frac{\alpha}{1-\alpha} - \frac{A}{1+A}\right)\log\frac{A'}{A} > 0 \; if \; \frac{\alpha}{1-\alpha} > \frac{N^o}{N^o+N^y}$$

A is the measure of aging $\frac{N^o}{N^y}$, so this condition is saying that the first effect is larger than the second as long as $\frac{\alpha}{1-\alpha} > \frac{N^o}{N^o+N^y}$. With a capital share of about 1/3 the ratio of the retired people need to be more than 50 percent for this condition to be violated, which is relatively far from being satisfied in the US data.
We report a more general formula in the footnote which illustrate that this condition is more likely to be satisfied the higher is σ. A higher value of this parameter will in general lead to further capital deepening.[8] As a numerical example, if the formula in the footnote is approximated around $\beta = R = 1$ then the inequality in (6) is satisfied as long as $\sigma > A = \frac{N^o}{N^y}$. For a value of 4, for example, that is not uncommon in the literature, then the ratio of old to young would need to exceed 4 in order for aging to be contractionary on output per capita.

The bottom-line, then, is that the empirical pattern observed in Figure 1, is predicted by a standard OLG model under various parameter configurations, even if one can think parameter configuration in which it does not apply.

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[7] The last equality sign is approximated around A'=A.
[8] More generally, for any σ, the condition is given by the following log linear approximation:

$$\tilde{y}'^{pc} - \tilde{y}^{pc} = \left[\frac{\alpha}{1-\alpha}\left(\frac{1+\beta_{R,\sigma}}{\frac{1}{\sigma}+\beta_{R,\sigma}}\right) - \frac{A}{1+A}\right]\log\frac{A'}{A}, where\; \beta_{R,\sigma} = \beta^{\frac{1}{\sigma}}R^{\frac{1}{\sigma}-1} = \beta^{\frac{1}{\sigma}}R^{\frac{1-\sigma}{\sigma}}$$

The key observation, however, is that the capital deepening requires the real interest rate to decline and the intensity of this effect depends on σ. In dynastic or representative agent models the real interest rate is fixed at β^{-1}, while here it is pinned down by the relative supply and demand for capital. In principle, there is nothing that says that real interest rate has to be positive (i.e. the gross rate R^k bigger than 1). This is precisely what the secular stagnation literature is all about. It says that *if* the real interest rate needed to make investment equal to savings is negative at full employment, and there are limit to which the interest rate can be adjusted, for example due to the zero-lower bound, the economy will experience a recession. Moreover, an aging society, i.e. one that has more old people relative to the working population, as we have seen, will in general need more interest rate adjustment to equate desired investment to savings.

2.b 2008-2016: Secular Stagnation

The fundamental mechanism that generates secular stagnation, is that the real interest rate cannot adjust to equate investment and savings at full employment. This is the sense in which it describes “excessive savings”. In order to capture this idea we need some reasons that prevent the real interest rate to fall enough. The most straight forward way of doing so is to introduce the zero-lower bound on the nominal interest rate, together with some additional assumptions we clarify shortly.

The way monetary policy is typically introduced, and a tradition we follow, is to assume that the government can issue paper currency and through that the central bank controls the short-term nominal interest rate, i_t, via open market operations in risk-free government short-term bonds. The price of this bond satisfies on the Euler equation

$$\frac{1}{C_t^y} = (1 + i_t)\beta E_t \frac{1}{C_{t+1}^o} \Pi_{t+1}^{-1}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is inflation, and P_t the price of the consumption goods in terms of money. Similarly, there is an arbitrage equation between the one period risk-free bond and the return on capital given by

$$(1 + i_t)E_t \frac{1}{C_{t+1}^o} \Pi_{t+1}^{-1} = \beta E_t \frac{1}{C_{t+1}^o} R_{t+1}^k$$

Adding these two pricing equations does not change the model we have already derived absent other assumption. It simply gives a theory of the price level once we add more detailed description of monetary and fiscal policy. The real interest rate, R_{t+1}^k, is the same as in the model analyzed in the last section, and so is output per capita and capital.

A theory of stagnation arises from the assumption that inflation cannot adjust freely. This allows monetary policy to directly affect the real interest rate, i.e. the return to capital, via the nominal interest rate and may prevent investment from matching savings at full employment. Recall that the reduction in the real interest rate was exactly key mechanism by which capital depending took place in response to aging in the previous section.

To illustrate this mechanism we take the path of least resistance. We simply *impose* that the *nominal wage is fixed* at some W_0 to start with, in which firms may not employ the entire labor endowment (so that labor is rationed equally across all workers). This may seem like an extreme assumption. Apart from illustrative purpose, we will obtain remarkably simple close form expressions, there are at least two reason for why this short-cut is worth taking rather than specifying a more elaborate dynamic wage and price schemes of which there are many. One reason is empirical, and this was Keynes original motivation. As a matter of fact, nominal wages simply do not tend to respond much to a rise in unemployment (for a recent example that documents this evidence, see for example in Schmitt-Grohe and Uribe (2016)). This has indeed been the experience in a number of countries during the Great Recession, to an extent the "missing deflation" associated with high unemployment was pronounced a theoretical mystery. The theoretical argument for this abstraction is more subtle but perhaps even more compelling. Assuming more flexible wage or price structure makes the drop in output at the zero bound in response to aging *stronger* rather than weaker, for reason first articulated by Fisher (1921), Tobin (1975) and De Long and Summers (1986). For a more recent treatment in the context of DSGE models see Bhattarai, Eggertsson, Schoenle (2014) who show that the mechanism about the destabilizing effect of wage/price flexibility always dominates at the ZLB. Consider an economy at the ZLB which "requires" negative real interest rate but cannot achieve it. Because the economy has "too high" real interest rate, relative to the natural rate of interest, there is output slack and expected deflation. Making prices more flexible, then, intensifies the expected deflation, thus increasing the real interest rate further, making the problem even worse and the output fall sharper. This, too, is the case in the current model, thus the assumption of perfectly fixed wage, paradoxically, is less extreme than the alternative of assuming something in-between fixed and flexible wages.[9]

The major implication of fixing the nominal wage rate, relative to last section, is that output is now *demand determined*. Below we consider a constant solution in which $i = 0$, i.e. again, we abstract from transition dynamics, which are not central to the point (but outlined in the Appendix), and focus instead on a stable secular stagnation equilibrium that can last for an arbitrary number of periods absent changes in the forcing variables. It is easiest to understand how output is determined by writing out aggregate spending as

$$Y = C + I + G$$

Again, it simplifies things a great deal to assume log utility, i.e., $\sigma = 1$. The consumption of the young and old can be derived to yield an aggregate consumption function

$$C = N^y C^y + N^o C^o = \frac{1}{1+\beta}(1-\alpha)Y \quad - \frac{1}{1+\beta} N^y \tau + \alpha Y \qquad = \left(\frac{1-\alpha}{1+\beta} + \alpha\right) Y \quad - \frac{1}{1+\beta} N^y \tau$$

We can use the first order condition of the representative firm with respect to capital to derive the demand for investment, yielding an aggregate investment function

[9] For completeness, we consider this case in the Appendix using a similar structure for downward rigid wage as in Eggertsson and Mehrotra (2014).

$$I = K = \frac{\alpha Y}{R^k}$$

At a superficial level the aggregate consumption function looks like an old fashion Keynesian consumption function in which aggregate demand depends upon a fraction of aggregate income net of the tax burden. Underlying it, however, is an intertemporal optimization problem, in which the labor income of the young is a fixed proportion $(1-\alpha)$ of total output.[10] Meanwhile the old consume all their income which is entirely derived from capital and thus in proportion α to total output Y. The investment function also looks old Keynesian. If the interest rate, i.e. the gross return on capital R^k declines, then the firms demand more capital for a given level of output Y. Putting the pieces together, and dividing by the total population, we now arrive at an aggregate demand in per capita terms given by

$$y^{AD} = \left(\frac{1-\alpha}{1+\beta} + \alpha\right) y^{pc} \quad + \frac{\alpha}{A}\frac{y^{pc}}{R^k} + \frac{\beta}{1+\beta} G^{pc}$$

where we have assumed that the budget is balanced in every period to substitute out for taxes. What we have written here is simply the spending for each agent in the economy, for a given level of production.[11] The consumer (young and old) will spend according to the first term, the firm capital expenditures are captured by the second, each derived from the respective maximization problems of the underlying agents. Observe that the steps we have taken have not required us to make any assumptions, as of yet, about the wage setting. The same equation applies in the model in the last section. If we replace each of the $y's$ with the flexible wage output derived in last section, this equation yields an expression for the implied real interest rate at flexible wages. The assumption of nominal frictions gives this equation a new life because it implies that the real interest rate cannot adjust to increase investment enough to match "desired savings". To be more specific, let us consider a secular stagnation equilibrium in which the nominal interest rate is zero, inflation is constant, so that $R^k = 1$ yielding

(6)

$$y^{AD} = \left(\frac{1-\alpha}{1+\beta} + \alpha + \frac{\alpha}{A}\right) y^{pc} \quad + \frac{\beta}{1+\beta} G^{pc}$$

which is plotted up in Figure 9 – an old but well known construction called the *Keynesian cross*.[12]

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[10] This follows from the assumption of perfectly competitive firms and Cobb-Douglas production. This implies a that output is split between output and capital in fixed shares.

[11] It is important here, that we assume that government spending, and thus taxes, is a fixed fraction of full employment output, see Appendix for details.

[12] The more general case that allows for movements in inflation is considered in the Appendix.

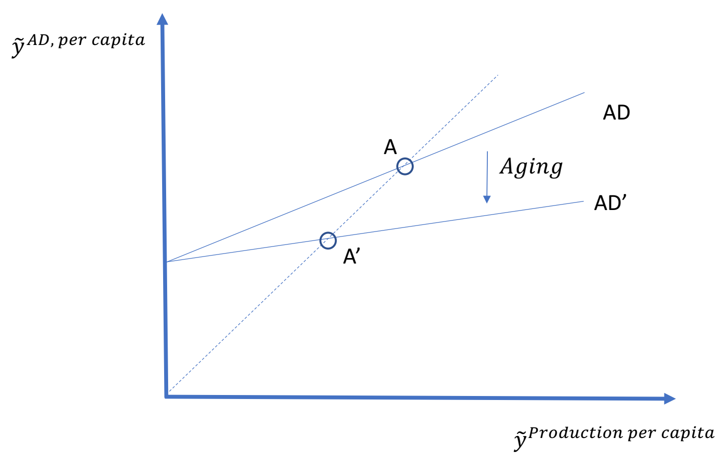


Figure 9 They Keynesian cross, aging and secular stagnation

The idea behind the Keynesian cross is to plot equation (6) as a *function* of any given level of production y^{pc} (this is the aggregate demand *function*). The amount of output demanded of consumption by consumers, and of capital by firms, as we have just seen, can directly be related to the aggregate production level. Thus, we can easily contemplate a situation in which there is a fictional “aggregate spending level” for any given production level in the economy, this is the AD function plotted in Figure 9. The 45 degree line, then, is the observation that in equilibrium *it must be the case* that aggregate spending implied this fictional production level in the AD function, has to be equal to the production itself so that $y^{AD,percapita} = y^{production\ per\ capita}$ representing a 45 degree line in Figure 1, a fixed point of the function AD(y^{pc}) $= y^{pc}$. This gives an equilibrium at point A.

This gives us a simple way of seeing the effect of aging in a secular stagnation, defined here as the situation in which the ZLB is binding and the economy finds itself on the Keynesian cross. We can see the effect of aging by directly inspecting how it changes the AD demand function. An increase in aging from A to A ‘makes the AD curve flatter, that is, there is now less demand for any given income level (production per capita).

What is the logic for this result? The key term in our characterization is how aging affects aggregate *investment demand*. We can express investment per capita, using the demand for capital by the firms, as

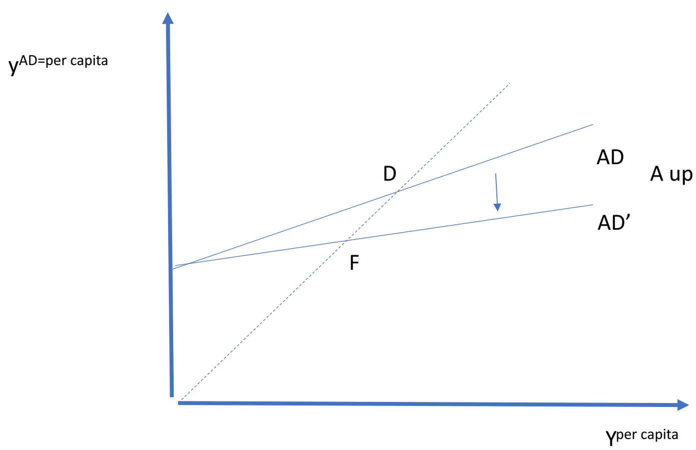
$$\frac{I}{N} = \frac{\alpha}{A}\frac{y^{pc}}{R^k}$$

Recall that before aggregate investment increased as *A* increased. This was because the increase in A was more than offset by a decline R^k. The firms responded to the decline in the interest rate by demanding more capital which in turn led to capital deepening in equilibrium. This link is now broken. The real interest rate is fixed, due to nominal rigidities, so there is no offsetting effect on investment via the interest rate reduction. Accordingly, investment declines. The result is a fall in aggregate production as shown at point A' in Figure 9. Observe that in a secular stagnation, therefore, the effect of aging on output per capita is *unambiguous*, i.e., it must decline. Doing a log-linear approximation as in last section we can show that aging has a negative effect on output, given by the formula

$$\tilde{y}_{\prime}^{pc} - \tilde{y}^{pc} = -\frac{\alpha A^{-1}}{1 - \frac{1-\alpha}{1+\beta} - \alpha} \log \frac{A'}{A} < 0$$

which is always negative, for the denominator is required to the positive for the secular stagnation equilibrium to exist.[13] This, then, explains the empirical patterns in Figure 1-6, the gist of which we summarize in a regression table in the next section.

Keynesian cross and aging



3. Regression results

Table 1 reports the simple correlations shown in Figures 1-6 with ordinary least square regression, using the data from Acemoglu and Restrepo (2017). The result represents regression of the change in (log) GDP per capital from 1990-2015 on our baseline measure of aging, the change in the ratio of the population of those above 65 to those between 20-65.[14] The baseline

[13] See Appendix for further discussion.

[14] Relative to their paper, we prefer to use as measure of aging the number of people above 65 years of age to the labor force, but their cutoff is instead 50 years of age. In the Appendix, we report the case in which the cutoff is 50, which does not materially affect the results. In our context we prefer above 65, because the main mechanism we are looking for has to do with

includes 169 countries. Table 1 reports OLS regression in changes (long differences) with robust standard errors. The first column show the first raw correlation we report which, as in Acemoglu and Restrepo is estimated to be positive, even if the uncertainty is large. Column (2) and (3) show that this positive relationship is driven by the data prior to 2008 rather than the period 2008-2015. Focusing on the period 1990-2008, the relationship is even more positive (column (2)), however, moving to the period 2008-2015, this correlation switches sign and becomes negative. Column (4) shows that this negative relationship appears to be largely driven by countries that where close to the zero bound in this period (this subsample is defined as the countries which had nominal interest rate at or below 0.5 percent at any point in this period).
In the Appendix we show in table A1, that the overall pattern is the same, if one uses instead the age cutoff in Acemoglu and Restrepo (2017) (see discussion in footnote 12).
Of these results the most interesting result, perhaps, is the negative correlation between aging and GDP growth reported in column (3) when several countries faced the zero bound. In Table A2 in the Appendix we explore the robustness of this negative correlation by adding the controls suggested by Acemoglu and Restrepo (2014) that include regional dummies, initial value of GDP and aging parameters, and so on (for further detail see Appendix). The bottom line, see first line column (4) in table A2, is that the negative correlation is still there once all the suggested controls are added, and the result is still statistically significant, even if the coefficient goes down in absolute value.

Table 1: Estimates of the impact of aging on GDP per capita from 1990 to 2015: old > 65 years

	(1) 1990-2015	(2) 1990-2008	(3) 2008-2015	(4) ≈ ZLB	(5) ≠ZLB
Change of the ratio of old to young	0.359	1.281	-1.949***	-1.941*	-0.907
	(0.753)	(1.117)	(0.514)	(1.040)	(0.800)
Constant	0.448***	0.339***	0.119***	0.0977***	0.110***
	(0.0450)	(0.0401)	(0.0154)	(0.0301)	(0.0214)
Observations	169	169	169	23	59
R-squared	0.002	0.011	0.061	0.195	0.016

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Conclusion

There has been an increasing attention of late about the effect of aging on GDP per capita. Researchers have noticed, however, a curious pattern. Looking over the last quarter of a century, it looks that in the cross section those countries experiencing aging have had higher GDP growth

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retirement, while their interest is ratio of older workers to younger workers, and the implied effect on automation, rather having anything to do with retirement.

per capita relative to those with younger population. In this paper, we suggest that a natural explanation for this is capital deepening associated with the worldwide fall in the real interest rate. We furthermore suggest, that this correlation is predicted to break down, and reverse itself, once the zero bound is reached, and point out that this is the key prediction of the secular stagnation hypothesis.

We do not wish to push country cross-correlations to far, for several reasons. The statistical power in our regression is not very strong. We do not think that is surprising, at least when considered in the context of the secular stagnation hypothesis. The genesis of the secular stagnation hypothesis has never been that aging is the only driving force between imbalances between desired investment and saving. Instead, it has been proposed as one of several candidates, including an increase in inequality, debt deleveraging, fall in relative price of investment, fall in productivity to mention but a few candidates.[15]

Finally, it is worth stating in few words what we think the secular stagnation hypothesis predicts and does not to predict. At its heart is the notion, that recessions at the ZLB can last for an arbitrary long time, and that there is no obvious adjustment mechanism back to normal. This of course does not imply that recessions at the ZLB need to last forever. It does imply, however, that factors that exaggerate savings and investment imbalances (where aging can be one of several contribution factor) make recessions at the ZLB worse than they otherwise might be. The data and model presented in the present paper are aimed at highlighting this general insight.

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[15] See e.g. Eggertsson, Mehrotra, and Robbins (2017) for a quantitative evaluation.

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Appendix

Algebra to be added

Additional Regression Tables

Table A1: Estimates of the impact of aging on GDP per capita from 1990 to 2014: old > 50 years

	(1) 1990-2014	(2) 1990-2008	(3) 2008-2014	(4) ≈ ZLB	(5) ≠ZLB
Change of the ratio of old to young	0.335 (0.210)	0.710** (0.291)	-0.529** (0.204)	-0.665 (0.455)	-0.213 (0.272)
Constant	0.420*** (0.0425)	0.324*** (0.0372)	0.129*** (0.0192)	0.106** (0.0405)	0.114*** (0.0258)
Observations	169	169	169	23	59
R-squared	0.013	0.032	0.038	0.139	0.009

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A2: Estimates of the impact of aging on GDP per capita from 2008 to 2014: old > 65 years

	SAMPLE OF ALL COUNTRIES					OECD COUNTRIES		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Change in the ratio of old to young (from 2008 to 2015)	-1.949*** (0.514)	-0.458 (0.567)	-0.417 (0.590)	-0.987* (0.592)	-0.552 (1.489)	-1.253* (0.713)	-1.074* (0.555)	-2.072* (1.246)
Initial GDP per worker		-0.0530*** (0.0128)	-0.0406*** (0.0144)	-0.0265 (0.0168)	-0.0281 (0.0176)		-0.0235 (0.0373)	-0.0188 (0.0341)
Constant	0.119*** (0.0154)	0.569*** (0.112)	0.476*** (0.115)	0.366*** (0.136)		0.0526* (0.0267)	0.365 (0.321)	

Observations	169	169	169	169	169	35	35	35
Differential trends by:							✓	✓
Population and initial age structure			✓	✓	✓			
Region				✓	✓			

Notes: The table presents long-differences estimates of the impact of aging on GDP per capita in constant dollars from the Penn World Tables for all countries (columns 1 to 5) and OECD countries (columns 6 to 8). Aging is defined as the change in the ratio of the population above 65 to the population between 20 and 64. Columns 5 and 8 present IV estimates in which we instrument aging using the birthrate in 1960, 1965, … , 1980. The bottom rows indicate additional controls included in the models but not reported: The population and age structure controls include the log of the population and the initial value of our aging measure. We report standard errors robust to heteroscedasticity in parentheses.

Algebra: To be added