From the Ergodic Hypothesis in Physics to the Ergodic Axiom in Economics

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Abstract

The direction of the mathematisation of predominant economics is unthinkable without the tacit underlying assumption of ergodicity. Despite its foundational character, the assumption of ergodicity is largely overlooked while discussing the intellectual history of the discipline. This is the cause why ergodicity is absent from the curriculum – although intimately intertwined with the equilibrium concept. Contrasted by such popular assumptions like rational expectations formation, representative agent, efficient markets, perfect competition, etc., that every student is aware of. Nevertheless, ergodicity is more fundamental than all of the mentioned assumptions taken together.

Ergodicity is a property of a mathematical system and originated from statistical mechanics, invoked by L. Boltzmann, who literally called it a 'trick' to simplify the needed mathematics. Ergodicity is fulfilled, if the time average of a system or process equals its ensemble average. However, if the time average of a system is different from its ensemble average, the system or process is called non-ergodic. The time average is the average of one observed trajectory or realisation of a process (one time series). The ensemble average is the average over every possible state of a system. Non-ergodicity is a necessary property of a mathematical model, if the model is supposed to describe occurences of endogenous novelties and change. The non-ergodic case is the more general, whereas the ergodic case is much easier to handle mathematically.

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Capitalistic economies are downright defined through their potential of evolution and innovation, and so are its very centerpiece financial markets. Accepting that proper mathematical models of economic or financial processes should posses the property of non-ergodicity, puts emphasis on the crucial role of time through which a certain amount of uncertainty enters into economic reasoning.

This contribution seeks to clarify this specific relation between the idea of (non-)ergodicity which is drawn from statistical mechanics and its role in and for economics and finance. The identified change of status is how the ergodic hypothesis in physics became the ergodic axiom in economics. Therefore, we follow the idea from rational mechanics of J. W. Gibbs via E. B. Wilson to P. A. Samuelson and eventually from the latter into rational economics. This methodological spillover from the natural sciences to economics and the assumption of ergodicity in particular enabled and shaped the mathematisation of economics seen since the 1940s.

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List of Abbreviations

CPI	consumer price index
DAX	German stock index
DN	deductive-nomological
EMH	efficient markets hypothesis
GDP	gross domestic product
HFT	high-frequency trading
IKE	Imperfect Knowledge Economics
SSB	spontaneous symmetry breaking
SSK	sociology of scientific knowledge

> In einem Fach, das mit einem derart schwierigen Gegenstand
> befasst ist wie die Ökonomik,
> ist ein Zustand der Ungewissheit nicht sehr angenehm,
> ein Zustand der Gewissheit aber ist bloß lächerlich.[1]
>
> Heinz D. Kurz (paraphrasing Voltaire)

> The primary task of economists is
> to be a prophylactic against popular fallacies.
>
> Henry Simons

1 Introduction

Time, like money, is *not* neutral to economic reality. During every economic crisis economic theory is questioned and sometimes this examination brings along new approaches, spurs the use of newly developed methods and turns the focus to novel issues. Since the financial crisis from 2007 and the subsequent economic crisis the non-neutrality of money is heavily discussed. However, the role of time is not. Albeit its paramount importance, the issue of *time* is underrepresented in economic theory and teaching, if at all present. Therefore, this article explores the role of time as a creative force in economic theory.

In his *Principles of Economics* Marshall identifies 'the element of time, the source of many of the greatest difficulties in economics'[2], and therefore sees it as central to economics. Resembling his comment on biology as the mecca of economics, he never incorporated a proper treatment of time in his economic theories. Also the famous and of course hotly debated normative definition of what economics should deal with by Robbins (1945), 'Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses'[3], contains a lesser-known statement about time on equal footing with the scarcity of resources: 'It is true that the scarcity of materials is one of the limitations of conduct. But the scarcity of our own time and the services of others is just as important'[4]. The first part of this definition, scarcity of materials, is being challenged since the beginning of the rise of digital goods.[5] As convincing as the evidence for digital goods may be, since humans can't eat digital goods, the basic question

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[1] Kurz, 2014, Preface, p. 5.
[2] Marshall, 1920, p. 76.
[3] Robbins, 1945, p. 16.
[4] Robbins, 1945, p. 21.
[5] Also recently by Rifkin, 2014.

amounts to whether or not mankind has overcome the scarcity of goods it needs to survive? This study is about the latter part, thinking about the role of time and accepting, that everybody is forced to make decisions in (his unique life) time and at an unique moment of time. Taking this seriously will be central in this article by means of dealing with non-ergodicity. Following Biervert and Held (1995, p. 12), different time dimensions, the arrow of time, uncertainty and subjectivity should become components of the research object of economics. This article is a contribution along these proposed lines.

Necessity of Meditating on Time for Economics

Beside price flexibility, the prerequisites for the existence of economic equilibria in a competitive economy are complete information or the existence of a complete set of markets for all future contingencies, whereby the latter was designed for replacing the former due to its known unrealisticness[6].[7] Only if economic agents have full information on the past, the present and most important the future (!), the reckoning of equilibrium prices can happen instantaneously.

Virtually, this is falling behind old insights of the known critiques insisting on the dispersed, personal and tacit nature of knowledge[8]. Ignoring them for a moment for the sake of the argument, there are two possibilities to answer the question: who actually owns this information and thus this knowledge?[9] It turns out, that either an auctioneer in the Walrasian sense or all actors could posses this knowledge. As already the original conception of complete information (especially about the future) is quixotic, both possibilies are as well. This shines a light on the relevance of time or – to be as distinct as possible at this point – the importance of the elimination of time for the existence of economic equilibria, the cornerstone for most of neoclassical economic theory.

In addition, the very recent developments in financial markets force the economics profession to become aware of the relevance of physical limits especially special relativity for economic theorizing.[10] Not only there exists a maximum speed at which even massless objects can move, also the speed of light c defines the maximum transmission speed for energy and information.[11] Even though latencies for financial transactions shrink, eventually they are approaching a fundamental

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[6] Arrow and Debreu, 1954.

[7] Also full contingency markets are not seen everywhere and thus are an unrealistic assumption as well.

[8] Von Hayek, 1945; Polanyi, [1958] 2008, 1966.

[9] A detailed discussion follows in chapter 4.

[10] Angel, 2014; Laughlin et al., 2014; Lloyd, 2000.

[11] Einstein (1905). In vacuum the speed of light amouts to $c = 299792458\frac{m}{s}$. It has to be mentioned, that the assumption of complete information is of course flawed since its first-time utilisation and at least since Einstein (1905). Its faultiness is independent but highlighted by recent insights.

limit. A minimum latency greater than zero implies

$$c < \infty \Rightarrow t_{latency}^{min} = \frac{distance}{velocity} = \frac{s}{v^{max}} = \frac{s}{c} > 0.$$

As 'our markets are now geographically diverse complex networks of competing trading platforms'[12], information can no longer be assumed to be available instanteneous for all involved parties (traders and exchanges) as assumed e.g. by the efficient markets hypothesis efficient markets hypothesis (EMH)[13], which violates concepts that are basic to finance theory like the *law of one price*[14]. Furthermore, economic theorizing and regulation is thereby confronted with seemingly insurmountable barriers in terms of fairness of market access and order execution, advantageous exploitating of arbitrage opportunities, consumer protection and market stability.[15] This by far outweighs (allegedly) increases in liquidity and efficiency through high-frequency trading (HFT). [16]

Structure of the Article

The reminder is organized as follows. Section 2 provides a new vantage point on the famous St. Petersburg Paradox, which was first achieved by Peters (2011). The careful evaluation of the paradox shows, how the use of ensembles or parallel universes contradicts its modelling framework. Concomitant, the original paradox undetectedly transformed from a problem in decision theory to a different problem in portfolio theory. This transformation is invalid for several reasons, especially if one vindicates the point of view of temporal naturalism as advocated by Smolin (2013a). To scrutinise this further, section 3 introduces the concepts of ensemble averages and time averages via a property of mathematical models referred to as (non-)ergodicity. I argue the case for the use of non-ergodic models in economics and finance. Elaborations on the ramifications of non-ergodicity for statistics and econometrics follow in some detail. Section 4 explores how the ergodic hypothesis from statistical mechanics could morph unnoticedly into the ergodic axiom in economics. Section 5 summarises the findings.

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[12] Angel, 2014, p. 279.

[13] Fama, 1970.

[14] Jevons (1888, p. 91) called it *law of indifference*.

[15] Angel, 2014.

[16] The concept underlying an economic theory, where equilibrium is defined following Arrow and Debreu (1954), corresponds to the block universe interpretation of special relativity. In a block universe interpretation everything (past, present, future) is fully determined and time does not exist. See Smolin (2013b).

> All experience shows that technological changes profoundly transform political and social relationships. Experience also shows that these transformations are not a priori predictable, and that most contemporary first guesses concerning them are wrong.[17]
>
> John von Neumann

2 The St. Petersburg Paradox Redux

In 1713 the Swiss mathematician Nicolaus I. Bernoulli posed in an exchange of letters with the French mathematician Pierre Rémond de Montmort the following problem[18]:

What is a reasonable price p for a ticket for the following lottery?

Lottery

1. On heads, the lottery pays €1, and the game ends. On tails, the coin is tossed again.
2. On heads, the lottery pays €2, and the game ends. On tails, the coin is tossed again.
3. On heads, the lottery pays €4, and the game ends. On tails, the coin is tossed again.
4. . . .

n. On heads, the lottery pays €2^{n-1}, and the game ends. On tails, the coin is tossed again.

. . .

Later N. Bernoulli's cousin Daniel Bernoulli worked on this problem, while he was appointed member of the academy of sciences in St. Petersburg and published a solution in the series of the academy, for which reason this lottery is called St. Petersburg problem[19].

2.1 How the St. Petersburg Problem became the St. Petersburg Paradox

To answer the question concerning a 'reasonable' ticket price, the usual consensus is to calculate the expected return of this lottery. This expected return is then thought of as a 'fair

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[17] Von Neumann Whitman, 1990, p. 3.
[18] De Montmort, 1713.
[19] Bernoulli, 1738, 1954.

price’.

$$
\begin{align}
E[\text{return}] = E[r] = E[\text{winnings} - \text{price}] &= E[\text{winnings}] - p \tag{1}\\
&= \sum_{n=1}^{\infty} \text{probability of scenario}_n \cdot \text{winning in scenario}_n - p \nonumber\\
&= \underbrace{\frac{1}{2}1}_{=\frac{1}{2}} + \underbrace{\frac{1}{4}2}_{=\frac{1}{2}} + \underbrace{\frac{1}{8}4}_{=\frac{1}{2}} + \ldots - p \tag{2}\\
&= \sum_{n=1}^{\infty} \frac{1}{2^n} 2^{n-1} - p = \sum_{n=1}^{\infty} \frac{1}{2} - p = \infty - p \tag{3}\\
E[r] &= \infty \tag{4}
\end{align}
$$

Equation 1 shows, how the expected return depends on the expectation of the difference between the winnings and the price, with the price p being a known constant.[20] Therefore, equation 2 particularises the composition of the expected winning, which is basically the sum of the winnings in every scenario weighted by the probability of the respective scenario. As every product in this sum in equation 3 equals $\frac{1}{2}$, the sum is diverging, hence equation 4.

If the expected return of a game is infinite, this is not only a strong incentive to play, it is as well reasonable to play independent of the ticket price. Because people are paradoxically not willing to pay any price for a lottery ticket, the St. Petersburg *problem* became the St. Petersburg *paradox*. Remember, following standard decision theory, an infinite expected return advises one to play no matter what ticket price.

More general, the basic structure of the paradox is: *What is the value of a lottery ticket, offering a tremendous profit with a very low chance?* Problems of this structure come up frequently in investment decisions.

2.2 St. Petersburg Paradox Seen from a Different Angle – Ensembles and Interaction with Parallel Universes?

Real participants in this lottery are not willing to pay more than a small amount, e. g. €5, for a lottery ticket. This hints at an incompleteness considering the calculation of the expected return, that seems to be covered by the intuition of people.[21] Therefore, let’s look at the calculation again, but now from the perspective of scenarios. What are the scenarios in the equations 1 - 4? They can be interpreted as every scenario being a different possible future state of the system or

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[20] Note that I use the word ‘winning’ and not ‘profit’, to indicate the origin of the used concepts, which originate from gambling, like most of early probability theory.

[21] There are, to the best of my knowledge, no experiments ever run offering the experimentees a St. Peterburg lottery. One obvious reason is, the supplier faces an unlimited downside risk of having to pay €2^{n-1}, in case the experimentee is lucky and achives a series of successive tail throws, equivalent to a high n.

a parallel world. Modal logic is providing a language for these parallel worlds. Possible, but not necessarily subsequent states, are called *contingent*[22]. Following the notion of different scenarios as parallel worlds, every different scenario is contingent, meaning it realises or could realise in a different parallel world. The set of all different scenarios or parallel worlds can be interpreted as an ensemble.

Definition 2.1. (Ensemble) *A set of independent and identical copies of scenarios or worlds or microconfigurations of a system is called an* ensemble*. The cardinality of the ensemble set can be infinite.*

The parallel worlds differ only in the winnings that realise in every world. If k of n worlds would not be different at all (realisation of the same winning), then they can be pooled to one world with its probablity adjusted weigth of $\frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n} = \frac{k}{n}$.

Another look at equation 1 from the perspective of an ensemble leads to the following reasoning:

$$E[r] = \overbrace{\underbrace{\frac{1}{2}1}_{\text{world}_1} + \underbrace{\frac{1}{4}2}_{\text{world}_2} + \underbrace{\frac{1}{8}4}_{\text{world}_3} + \ldots + \underbrace{\frac{1}{2^{100}}2^{99}}_{\text{(lucky) world}_{100}} + \ldots}^{\text{ensemble of all possible parallel worlds}} - p \tag{5}$$

$$= \sum_{n=1}^{\infty} \text{weight}_n \cdot \text{winning in world}_n - p \tag{6}$$

$$= \text{ensemble average} - p \tag{7}$$

What the calculation of the expected return actually computes, is an average expected winning over the ensemble of all possible worlds. This is called an ensemble average, in this case an ensemble average of the expected winning. Or put differently, the expected winning is computed as an ensemble average. As one will see, it is this particular way of computing an average as an ensemble average, that lies at the foundation of the paradox.

2.2.1 Forbidden Interaction in the Ensemble

The key point of this article is missed easily, because the following may appear remote from traditional economic reasoning. It helps to be precise on what is actually written in equations 1 - 7. $E[r]$ is calculated, as if there is an interaction within parallel worlds. $E[r]$ is calculated, as if the payoff from one lucky world could compensate for all other worlds below the break-even point. Thus, a former problem in the realm of decision theory is treated, as if it were

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[22] Garson, 2013.

a mere portfolio problem. As if any participant in the lottery could split his ticket to buy at least one lucky world besides all the other unfortunate worlds. To be distinct, this clearly is a violation of space and time, in which the original problem was proposed. The use of the ensemble though may seem elegant, but can't be a proper solution to the paradox at least for two several reasons.

1. First, we live in time! Which means, we live in just one universe. If some other copy of ourselves in a parallel universe wins a huge amount (because his world is for example lucky world$_{100}$) is of no importance to us, since we can not interact with this copy of ourselves and ask him to share his/our money. Therefore, we want to know, whether we should partake in the lottery several times in a row in one and the same universe.
2. Second, if we use the concept of a multiverse, then we also have to accept the no-interaction premise between parallel worlds.

To put the St. Petersburg paradox in such a setting, links it to interesting discussions in theory of science, physics and cosmology, which are of high relevance for economics. The role of the reality of time in scientific or economic explanations is discussed in Mangabeira Unger and Smolin (2015) and Smolin (2009, 2013a,b), in which the philosophical stance of temporal naturalism takes time seriously. That the concept of an ensemble or multiverse is uncommon for most economists is shown by the fact, that it is used unconsciously and only rarely to nowhere made explicit in the economics and decision theory literature (not only of the St. Petersburg paradox). The many-world interpretation of quantum mechanics is one alternative explanation to the famous Copenhagen interpretation, that gained renewed interest in the last decades[23]

2.2.2 Finite Ensemble of Parallel Worlds

If we assume the cardinality of the ensemble to be infinite, as in equations 1 − 7, say infinitely many parallel worlds, it is immediately visible, no matter how high a ticket price p may be, it can't exceed a diverging sum, which consists of infinite terms in the sum (equations 3 or 7). However, in almost all real-world situations the ensemble is a finite set, eventually because there is a limited amount of money on the planet. Not only for theoretical reasons it is interesting to investigate the paradox with infinite cardinality of the ensemble set, also for pragmatic reasons regarding giving advice in decision situations.

In case of the St. Petersburg paradox, it could be simplified in assuming the ensemble being a finite set, not only because there is a limited amount of money on the planet, which means, both the provider of the lottery and the gambler, accepting the wager, posses only a finite amount of money. Hence, the provider can not afford to pay every winnings possible, and the gambler

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[23] Everett III, 1957, 1973; Tegmark, 2003, 2007.

cannot afford every ticket price. A provider of this lottery, say a casino, would of course charge a huge ticket price, hence, not everybody could partake. But if willing participants could borrow short term, they can always argue to the creditor in their own favor with the unlimited expected return and therefore should always obtain credit.

If we assume an ensemble of finite cardinality m, this is equivalent to a finite possible wealth of the provider. E.g. in the luckiest world m consecutive tail throws take place leading to a maximal winning of €2^{m-1}. Besides, a finite ensemble set is conceivable, as m could be a maximum wager the provider is willing to stake. Or the maximum wealth the gambler believes the supplier of the lottery owns, because the supplier has an incentive to offer the bet for a ticket price that matches his wealth level.

2.3 Remark to Former Solution Attempts to the St. Petersburg Paradox

Throughout the history of economics different attempts to solve the St. Petersburg paradox emerged, among others most famously the introduction of bounded utility functions via logarithmic utility[24] or square root utility as proposed by G. Cramer.[25] This section won't give a survey on solution attempts, as all of them do not tackle the main problem lying at the foundation: the interaction between parallel worlds via the computation of an ensemble average. Bounded utility functions are nothing but a pseudo-solution as they replace the absolute money value through the psychological concept of utility. If this utility function is constructed to be bounded by any concave function, this function assigns lower values to high winnings in the infinite ensemble set than the absolute value of money would. As Peters (2011, p. 4926) alluded '[t]his is problematic because the arbitrariness of utility can be abused to justify reckless behaviour, and it ignores the fundamental physical limits, given by time irreversibility, to what can be considered reasonable [. . .] arbitrary utility functions are replaced by the physical truth that time cannot be reversed.'[26] Therefore, this paper takes time seriously.

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[24] Bernoulli, 1738.

[25] As cited in Samuelson (1977).

[26] Additionally, this illustrates how easy rationality in economics gets misunderstand, see Geisendorf, 2009, 2010.

> Science is often characterized as a quest for truth,
> where truth is something absolute,
> which exists outside of the observer.
> But I view science more as a quest for understanding,
> where the understanding is that of the observer,
> the scientist. Such understanding is best gained by studying
> relations – relations between different ideas,
> relations between different phenomena;
> relations between ideas and phenomena.[27]
>
> Robert Aumann

3 (Non-)Ergodicity

3.1 The Origin of Ergodicity

In 1884 the physicist Ludwig Boltzmann first introduced a property he called ergodicity[28] to describe a closed thermodynamical system in precise mathematical terms. A closed thermodynamical system exchanges no energy with its environment.[29] Important is, the system is analysed by an external observer, for example a closed box of gas molecules is studied by a scientist. Additionally, assuming no dependence of the system state to its initial conditions, simplifies the mathematical description of the state of such a system a lot. Because many of the forces, that could potentially influence the state of the system, are thus assumed away. The final state, where the system comes to rest, also called its thermodynamical equilibrium or state of maximum entropy, is solely defined by the properties of the gas molecules like their velocities and positions in phase space.

That is why Boltzmann (1884, p. 123) himself called the introduction of the ergodicity property simply a mathematical trick, in the German original literally a *Kunstgriff*. Therefore, ergodicity is defined in the following way.

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[27] **\cite {Hart2005}**.

[28] έργον [*ergon*] for work and οδός in modern Greek or ὁδός [*hodos*] in ancient Greek for way or path. Trajectories need to move through the whole phase space, which is of course highly questionably to realise empirically.

[29] From the first law of thermodynamics follows no heat transfer and conservation of energy.

Definition 3.1. (Ergodicity) *A system is ergodic, if the following equalisation is valid*

$$\begin{aligned} \textit{Time Average} &= \textit{Ensemble Average} \\ f^*(x) &= \langle f \rangle \\ \lim_{T\to\infty} \frac{1}{T} \int_0^T f(\phi_t(x))\mathrm{d}t &= \frac{\int_S f(x)\mu(\mathrm{d}x)}{\mu(S)} \end{aligned} \tag{8}$$

with S being the energy surface of that system in phase space, ϕ_t the state of the system at time t, μ the measure, which describes the volume in phase space and f an arbitrary defined μ-integrable real valued function.[30]

Ergodicity creates a relation between the behavior through time and the behavior in all possible states of the system. The set of all possible states of a system build the phase space. The state of a system (at an arbitrary point in time) is characterised by its location in phase space. If one thinks of all other possible realisations at a given point in time, one can think of them as parallel worlds. Simplified, ergodicity relates time to the phase space.

Time Averages On the left hand side of the equation 8 is the time average of the process, that is simply the average over a certain realisation of one process.

Ensemble Averages The right hand side of the equation 8 is the ensemble average of the process, meaning the average over all possible states of the system at a certain point in time.

3.2 The Classic Example of Throwing a Die

In the following section the ergodicity, i.e. equivalence of ensemble average and time average, is illustrated by checking it experimentally for the simple example of throwing a die.[31] We know nothing about the die and the goal is to obtain the unknown probability that an arbitrary face appears after throwing it.[32] The experimenter has two ways to reduce the randomness of his measurements.

Reduction of Randomness Using the Ensemble Average The experimenter lets n experimentees each throw a die once. Then he checks, how many times k did an arbitrary face show up,

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[30] Based on Lebowitz and Penrose (1973, pp. 2).

[31] This experiment is described in Peters and Mauboussin (2012).

[32] Even the number of faces could be unknown and could be discovered by means of the experiment.

e.g. how many 6s were thrown? With an increasing number of experimentees the randomness gets more and more removed,

$$\lim_{n\to\infty} \frac{k}{n} \to \frac{1}{6} = P(X = 6),$$

thereby he is calculating an ensemble average of all throws.

Reduction of Randomness Using the Time Average The experimenter lets one experimentee throw a die t times. Then he checks, how many times k did an arbitrary face show up, e.g. again how many 6s were thrown? With an increasing number of throws of the one experimentee the randomness gets more and more removed,

$$\lim_{t\to\infty} \frac{k}{t} \to \frac{1}{6} = P(X = 6),$$

thereby he is calculating a time average of the throwing sequence.

As it turns out the ensemble average equals the time average for every face that could possibly show up. Hence, a correct mathematical model of the whole die turns out to be ergodic. There exists an unique measure, namely the probability mass function P and for face i reads

$$Pr(X = i) = \frac{k_i}{n} = \frac{k_i}{t}.$$

For a regular die having six faces this comes down to

$$Pr(X = i) = \frac{1}{6}.$$

3.3 Elimination of Time

The assumption of ergodicity necessary for the existence of an equilibrium enables the scientist to make a statement about the system behavior without having to observe all possible realisations of system states. Just one trajectory is enough to infer all the future behavior, at least probabilistically. Eventually, this brings along the elimination of time from scientific descriptions of the world, time is ‘integrated out’, as Weintraub (1991, p. 102) describes this paradoxically procedure:

> ‘Equilibrium’ is then interpreted as the limit of the dynamic behavior of the system. That is, a solution of the dynamic system involves time, so as time is allowed to pass out of the picture, as it were, or is **integrated out** by a limiting process, or if we wait until time is no longer meaningful to the statement of the problem, $\lim_{t\to\infty} x_i(t) = \bar{x}$, the equilibrium. (bold emphasis added by MK)

Hence, ergodicity allows to infer the system behavior from one observed trajectory (in physical terms) or in more economic terms from one observed time series. Hence, following the conceptual understanding of ergodic processes, there is no need to study the history of a process, because there is no sensitivity to initial conditions, 'in an ergodic system time is irrelevant and has no direction'[33]. The elimination of time is what is most often swept under the carpet of equilibrium analysis, if an economic equilibrium is defined via the physical metaphor of an equilibrating mechanical rest point. In this wording lies some confusing potential, as is also noticed by NIEHANS (1997, p. 58), '[i]n der Nationalökonomie wird die Lösung eines solchen [Gleichungs]Systems oft ein »Gleichgewicht« genannt, obgleich die physikalischen oder normativen Konnotationen dieser Bezeichnung durchaus irreführend sein können.' This is not dramatic, in cases where it simplifies the analysis a lot, but the price that has to be paid for this simplification is the elimination of time. A careful analysis whether this feature could also be a bug, is seldom carried out. Hence, mathematisation is so often solely seen as a triumph of science and an achievement per se, thereby leaving an important part, namely time, unconsidered.

3.4 Contingency Space

It is important to state, that the phase space concept, which is used to calculate the ensemble average which in turn replaces the time average, is a timeless mathematical structure. What is more relevant for all problems, where the arrow of time is at work, is another abstract structure something that I call the *contingency space*.

Definition 3.2. (Contingency Space) *A contingency space is a series of phase spaces over time. Over time, new regions of the phase space can be accessible, totally new regions are developed or old regions vanish from t_1 to t_2, with $t_1 < t_2$.*

Contingency space in this context describes the space of possible other states of the system at a given point in time and, more importantly, also in the future. Hence, the contingency space consists of all states, a system can be imagined in, over time. This is ultimately the only abstract structure, in which thinking about evolving systems is sensible, if we recall, that evolving systems are inherently characterised by endogenous novelty or innovation. Thus, the contingency space is the sum of all possible phase spaces over time. This hyper phase space can only be known to an entity with complete knowledge of everything in the past and more contradictory in the future. This constitutes the well known 'problem of the utilization of knowledge not given to anyone in its totality', pronounced so often by VON HAYEK (1945, p. 520).[34] If one knows the phase space, one knows a lot about the system. Especially for forecasting purposes, one often implicitly assumes

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[33] PETERS and MAUBOUSSIN, 2012.

[34] Also mentioned in VON HAYEK ([1974] 1989, p. 4): 'Into the determination of these prices and wages there will enter the effects of particular information possessed by every one of the participants in the market process – a sum of facts which in their totality cannot be known to the scientific observer, or to any other single brain.'

to know the whole phase space at future instants (known as perfect foresight) or assumes it to stay constant (timeless entity).

Originated from modal logic, *contingent* means possible but not necessary. A contingent state of a system is possible, but another contingent state can finally realise. Imagine the simple contingent movement of an asset price p from time t to time $t + 1$. If we simplify the future states at $t + 1$ to the following three $\{\text{up, unaltered, down}\}$, then surely we will observe only one realisation in $t + 1$. Either the asset price p_{t+1} went up ($p_{t+1} > p_t$), didn't change ($p_{t+1} = p_t$) or went down ($p_{t+1} < p_t$). We won't observe an asset, that at the same time went up and down. In thinking about the realisation of something as the average of everything possible, we are endowing SCHRÖDINGER's cat with an asset[35]. This is no more reasonable in economics than it is in physics, where a lot of passionate discussion arise exactly at the point of how can one tangible classical (macro) state be explained by many intangible quantum states.

The difference between the contingency space and phase space becomes relevant, whenever the phase space is evolving over time or, put differently, whenever novelty appears. The contingency space is a describtion of the evolution of the phase space over time of a system or the phase space with a time index. If the phase space is not changing over time, then it may be possible to use ergodic mathematical techniques like statistics to analyse the system. This does not imply, that the mathematical model of the system will turn out to be ergodic!

The contingency space of systems an economist, a biologist, a historian or anybody, who studies living objects, is interested in, is not only made up of permutational many states, most of them are not foreseeable. These infinitely many and unforeseeable future states of such systems can't be analysed assuming ergodicity, because it becomes impossible to average over the ensemble, if already the ensemble elements or not known. A possible equality of ensemble average and time average may be undecidable in this case, hence linking non-ergodicity and undecidability[36].

KAUFFMAN (2000) coined the term the *adjacent possible* and discussed the role played by the adjacent possible in the evolution of life. He analyses, how molecules form amino acids and other compounds through accessible regions in the contingency space of all chemical stable compounds. Earlier in KAUFFMAN (1993, pp. 181) the following statements are found, which especially discuss the role of non-ergodicity for the evolution of life:

> For a system released at a point in phase space, the flow under the drive of collisions is an ergodic wandering over phase space. Because the **system conserves energy**, the phase volume occupied by a flowing cloud of points, representing the system started at neighboring points in phase space, **remains constant as it flows. From this constancy**

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[35] SCHRÖDINGER, 1935.

[36] A more detailed account of the relationship between undecidability and non-ergodicity is along the way and will naturally add to subsection 4.3.

> **and ergodicity**, it becomes possible to calculate the probability that the system is in any specific volume of phase corresponding to some macroscopic state, such as all particles being in a corner of the box.
> [. . .] **Second, unlike closed physical systems, which conserve energy, biological systems are open thermodynamically, typically dissipate energy, and have attractors. Third, the regular unfolding of ontogeny alone suffices to say that biological systems cannot wander randomly and ergodically over their space of possibilities.** The essence of development from the fertilized egg is its astounding combination of complexity and utter regularity. **Fourth, the most profoundly random aspect of biological systems is random mutation in the** ***space of possible systems***. That is, evolution is an adaptive, or drifting, process which searches across the space of biological systems. **Thus unlike statistical mechanics, which can be characterized as a more or less ergodic flow within the state space or phase space of a single system, evolution is a more or less adaptive flow across a space of systems.** (bold emphasis added by MK)

Thoughts very similar to a contingency space, can be found in Northrop (1941, pp. 11), 'total volume of economic wants does not remain constant with time.' Comparing the economic method with that of Newtonian physics, the constant flux of people's wants and desires hinder the construction of any conservation laws in economic theory of his time, which are necessary for the construction of a theory of economic dynamics, eventually for prediction purposes,

> Let us now consider the fundamental concept of contemporary economic theory with respect to Newtonian mechanics. This concept is 'economic wants'. It is quite clear that the total volume of these wants does not remain constant through time. They are in constant flux. In short, the basic subject matter of economic science, as conceived by contemporary economic theory, does not have the property of obeying a law of conservation.

A similar remark already made by F. H. Knight 'There is nothing in economics corresponding to either momentum or energy, or their conservation principles in mechanics [. . .] there is no definable economic space'[37]. The lack of conservation laws in economics became later a central theme in Mirowski (1991). As all these citations show, the use of ergodic methods in economics may be a bit to hasty, if one can not state a conservation law backing up their application. Thus, the next section discusses multiple ramification of non-ergodicity.

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[37] Knight, 1921, p. xxiii.

3.5 What is Non-ergodicity?

Non-ergodicity is the case, if equation 8 turns into an inequality.

Definition 3.3. (Non-ergodicity) *A system is non-ergodic, if the following inequality holds*

$$\begin{aligned} \textit{Time Average} &\neq \textit{Ensemble Average} \\ f^*(x) &\neq \langle f \rangle \\ \lim_{T\to\infty} \frac{1}{T}\int_0^T f(\phi_t(x))\mathrm{d}t &\neq \frac{\int_S f(x)\mu(\mathrm{d}x)}{\mu(S)}. \end{aligned} \tag{9}$$

3.5.1 Stationarity & Ergodicity

The following example is constructed to show, that stationarity of a time series is not a sufficient criterion for the ergodicity of the underlying process.

Example 3.5.1. (Breakdown of Ergodicity)[38] *Let x be a binary random variable with $x \in \{0,1\}$ and $x_t, t \in \mathbb{N}$ the corresponding stochastic process. Let there be full stochastic dependence between any x_t and x_{t+1}, such that ultimately the first random variable determinates the path of the stochastic process to be either a sequence of only 1s $(1,1,\ldots)$ or 0s $(0,0,\ldots)$. Due to stochastic dependence the stochastic process is stationary. The ensemble average will yield $\frac{1}{2}$, which will not coincide with any value of the time average, which will be either 1 or 0. Hence, the stochastic process is stationary, but not ergodic. Over time the process will not cover the whole state space.*

3.5.2 Yet another Critique Point on Metrics

Since the debate about the usefulness and soundness of econometrics started parallel to the rise of the mathematization of economics in the 1930s, for example with KEYNES' critique on the TINBERGEN method[39], it is still not settled and of urging topicality. Now that a large part of economists all over the world is mainly focused on building (econometric) models to find the 'true' (presumed to statically exist) data generating process or causal structure[40], it is especially important to understand, what heavy loaded assumption the ergodic hypothesis is.

The LUCAS critique of 1976 seen from the non-ergodicity perspective teaches us, that the use of econometrics and probabilities in economics and finance especially as a mean for economic policy decisions, can only end in an arms race between the econometrician/politician and the system. The

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[38] This example was first presented to me by Prof. STEFAN HUSCHENS.

[39] KEYNES, 1939, 1940; TINBERGEN, 1940.

[40] And willingly applying this 'insight' in policy counselling, if asked for.

economist is trying to steer the economy and the economic sytem under investigation. The system is continually or rather often discontinually in a nonlinear manner changing its internal structure as a reaction to the econometric measurement or observation process, in striking resemblance of that famous observer effect, that is invoked in quantum measurements.

3.6 Dynamics

Many scientific theories share the use of the concepts or at least the word 'dynamic'. It is important to draw a distinction between two different kind of dynamics, called LAPLACIAN and evolutionary in this paper.

3.6.1 Laplacian Deterministic Dynamics

Most dynamics incorporated into scientific theories are deterministic. This means, there may be change over time, but the nature of this change is constant and known. The LAPLACIAN demon is a famous metaphor for this situation. If a god-like superhuman creature could know the initial conditions of all particles in universe with infinite precision and the laws of motion, it could predict the future behavior of all particles. So to speak, the dynamics of the dynamics is not dynamic or there is no higher order dynamics.[41] More formally, the second derivative of the changing parameter, let it be x, is equal to zero,

$$\frac{\partial^2 x}{\partial^2 t} = 0.$$

Even the dynamics in quantum physics is deterministic, 'According to the Schrödinger equation, this wavefunction evolves over time in a deterministic fashion that mathematicians term 'unitary'. Although quantum mechanics is often described as inherently random and uncertain, there is nothing random or uncertain about the way the wavefunction evolves.'[42] Of course, LAPLACIAN dynamics exhibits an arrow of time, but time is still seen as an impediment to theory construction and ultimately an illusion and not as a creative force, that plays an important role in shaping the outcome. From a HAYEKIAN perspective it is highly questionable, how any single entity could gather all the relevant information to be an actual LAPLACIAN demon[43]. Furthermore, how could one falsify the existence of such an entity?

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[41] That's why it is questionable, whether it still deserves the term 'dynamics', if eventually one derivative will equal zero.

[42] TEGMARK, 2007, p. 23.

[43] VON HAYEK, 1945.

3.6.2 Evolutionary Dynamics, Evolutionary Development

A more interesting kind of dynamics arises, when the dynamics is non-ergodic. Hence, when higher derivatives of the changing parameters are never zero.

Definition 3.4. (Evolutionary Dynamics) *The dynamics is called an evolutionary dynamics, if for every partial derivative of order k*

$$\frac{\partial^k x}{\partial t^k} \neq 0 \tag{10}$$

or if there are different partial derivatives at different times for $n \in \mathbb{N}, \tau = t + n$

$$\frac{\partial^k x}{\partial t^k} \neq \frac{\partial^k x}{\partial \tau^k}, \tag{11}$$

and no limiting value for

$$\lim \left(\frac{\partial^k x}{\partial \tau^k} - \frac{\partial^k x}{\partial t^k} \right) (\neq 0) \tag{12}$$

exists.

This kind of dynamics is central to evolutionary economics, hence the wording.[44]

Evolutionary economics stresses the fact, that there is no way to express evolutionary dynamics through an algorithm. In other words, it's impossible to model evolutionary dynamics arithmomorphically, 'Neues entsteht dann, wenn sich die *Handlungsbedingungen und -möglichkeiten* von Wirtschaftssubjekten so verändern, daß dies nicht durch einen Algorithmus auf arithmomorphem Weg abgebildet und prognostiziert werden kann'[45]. What this quotation also highlights, is the connection between evolutionary dynamics and the potential of novelty. Hence, evolutionary sciences declare parts of their field and dynamics as not being amenable to being modelled in an ergodic or even mathematical formal way by definition. This raises interesting methodological question adressed in section 4.3.

3.7 Ramifications of Non-Ergodicity for the Role of Time and Causality

Non-ergodicity invokes time as a creative force. As is shown in the following, the existence of time raises deep questions regarding the existence of causality.

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[44] In the literature on evolutionary game theory the term 'evolutionary dynamics' is also used, e.g. in Nowak and Sigmund (2004), but describes something orthogonal to what is discussed here.

[45] Lehmann-Waffenschmidt, 1995, p. 112.

3.7.1 Analytical vs Historical Time

Figure 3.7.1 reveales the difference between an analytical understanding of time in the ergodic case and an historical understanding of time in the non-ergodic case. The left part of fig. 3.7.1 shows the ergodic case. Here, the present moment is merely an arbitrary dividing line between an otherwise homogeneous medium, therefore the straight vertical line. Homogeneous in the sense of there is nothing that distinguishes the past from the future, e.g. the probability distribution of a certain random variable is the same before, during and after the present moment. That is why Davidson (2009, p. 328) famously described this sitauation as 'the future is merely the statistical shadow of the past', which is why instead of the future taking place there is just a mirror image of the past taking place. For what reason the concept of time used in the ergodic case is called an analytical time, because it is just something like an index or a counter and nothing more. Time in this conception is fully reversible.

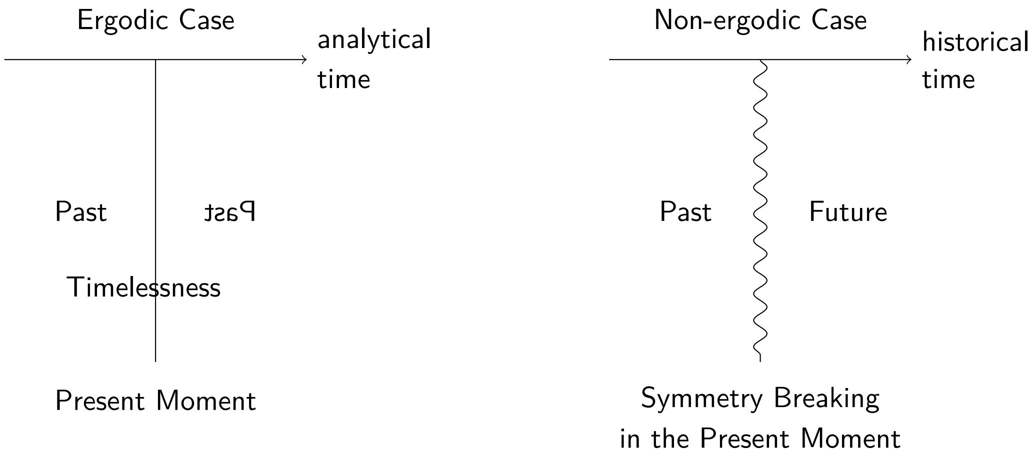

Figure 1: Left: Time in the ergodic case. Probabilities (drawn from the past) are meaningful for the future 'the future is merely the statistical shadow of the past.'
Right: Time in the non-ergodic case. As probabilities can't be drawn from the future, they are 'meaningless' for the future.

Quite the contrary in the non-ergodic case on the right of fig. 3.7.1. In the non-ergodic case the present moment carries the potential of a symmetry break taking place distinguishing the past from the future, indicated by the snaky vertical line. This symmetry break of the system or the process occurs in such a way, that the past is clearly different from the future. Hence, the future is not anymore just a mirror image of the past, but owns its unique new characteristics, e.g. a probability distribution of a certain random variable that is different after the present moment than it has been before it. The concept of time in a non-ergodic case is therefore called historical or real time.

The symmetry break described here is quite similar to the explanation of certain physical phenomena via spontaneous symmetry breaking (SSB) in Liu (2003). Also a connection between

SSB and non-ergodic systems is made in Liu (2003, p. 599): 'few systems, if any real ones, are proven ergodic and some are proven non-ergodic, and even for those proven ones, how does ergodicity gives us any reason to believe the probability it gives for finding a single outcome at a certain moment? [. . .] SSB systems are obviously nonergodic.'

3.7.2 'Ergodic' Fallacy or Causality Fallacy

Now all the vocabulary is introduced to understand the ergodic fallacy. The ergodic fallacy is comprised of the misbelief in a causal relationship, when the relationship is either not existent or constantly changing. The latter case proves most interesting, as it leads to questions like 'What is causality? Can causality change? If causality can change, is it then the same causality or something different after it changed?, and finally, Does causality exist at all? What is its ontological status?' Broadly speaking, the ergodic fallacy is the search for (natural) laws if there can't be such laws.

To elucidate the ergodic fallacy, the meaning of fig. 2 is easily disentangled. First, it is accepted to be embedded in an environment where time is real, thus the concept of historical time. In such an environment, for example a scientist observes a process in a finite time window, the observation window (1). The scientist has no knowledge or data over any time periods before that window or is simply not interested in these time periods, so everything before that observation window is just 'not analysed'. A scientist falling prey to the ergodic fallacy, now believes it is possible to inductively infer a '(spurious) causal relationship' (1) from his observed time period into the future. But the present moment marks the end of his observation period and carries the potential of non-routine change[46] If in a present moment[47] a symmetry break occurs, the scientist is confronted with situation, in which there is one causal relationship before the present moment and a different one after it. This is called a '(2) non-causal window' in fig. 2. It is possible that the occurence of a symmetry break is not recognised immediately after it happened and therefore the causal relationship seems to hold for while, indicated by the mirror image of the word 'Ergodicity' after the first symmetry break. Eventually, the causal relationship breaks down, 'something new' is taking place and destroys the mirror imgae character of the future seen from the present moment or the past (3). Of course, there can be successive symmetry breaks, creating successive '(4) non-causal windows'. Furthermore, a time window which spans equal or more than one symmetry break is a non-causal window, non-causal in the sense of changing causal relationship.

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[46] To use the wording of Imperfect Knowledge Economics (IKE) by Frydman and Goldberg, 2007, 2011 for unpredictable change, which describes exactly the same as what is meant here.

[47] In philosophy of time the present moment is often called 'The Now'.

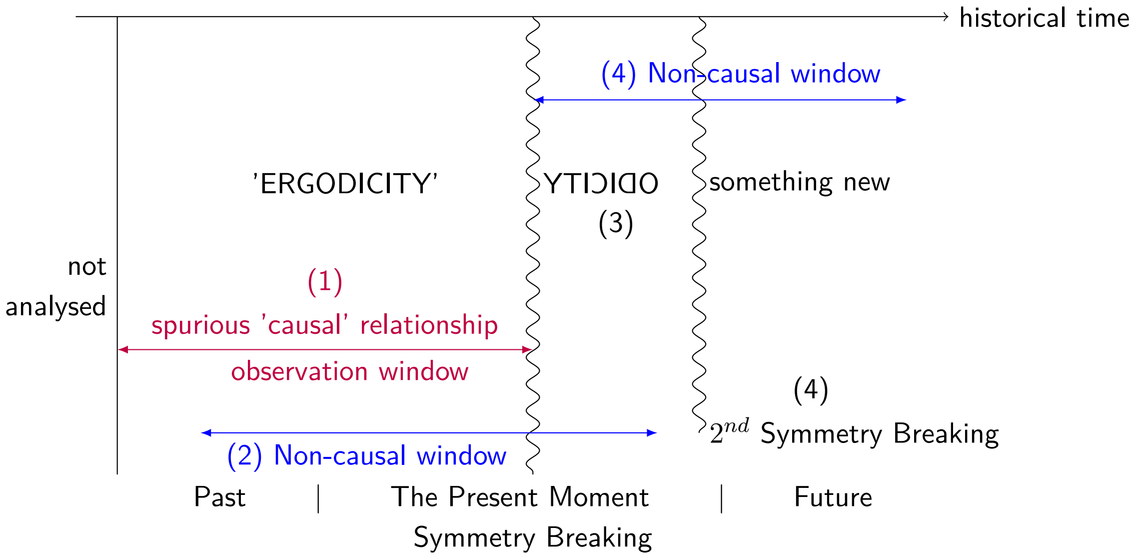

Figure 2: Ergodic Fallacy or Causality Fallacy.

3.7.3 Potential Drivers of Symmetry Breaks

This amounts to the interesting question of what are the drivers of symmetry breaks in general and in economics specifically. The first candidate that comes into an economist's mind is innovation. Innovation economics is a rich source on explanations how innovations once they occured diffuse or don't, but not so much on what drives these innovations other than entrepreneurial creativity. I want to concentrate here on a more fundamental driver of symmetry breaks in my opinion, namely, the process of observation itself. Imagine if science itself, the pure act of cognition, is changing the system in unforeseeable ways? This creates a view of science, which destroys the stability of its research subject. Science inhibits then what is seen by many as its final success – the discovery of ultimate truths, that reside outside of time like a mathematical object (triangle, line, point, etc.) or a natural law, which is often written in formal mathematical language.

There are several insights, some of them made already some decades ago, which basically share the same core of an evolving system that changes its inner causal structure in unforeseeable non-deterministic ways. Among them are the Lucas critique, Soros' reflexivity, Goodhart's law or Wiener's strong coupling.[48]

The more fundamental question this raises is the one about the appropriateness of any timeless (mathematical) structure to describe time-bound (economic) phenomena? This topic will be adressed either in a later section on the philosophy of (non-)ergodic economics or a second paper.

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[48] Goodhart, 1975; Lucas, 1976; Soros, 2013; Wiener, [1948] 1985.

> When you cannot express it in numbers,
> your knowledge is of a meagre and unsatisfactory kind.
>
> LORD KELVIN

> If we cannot measure a thing,
> go ahead and measure it anyway.
>
> FRANK H. KNIGHT

> Yes, and when we can express it in numbers,
> our knowledge will still be
> of a meagre and unsatisfactory kind.[49]
>
> JACOB VINER

4 From the Ergodic Hypothesis in Physics to the Ergodic Axiom in Economics

Structure of this section

This section shows how the status of the ergodic *hypothesis* central to statistical mechanics undetectedly morphed into the ergodic *axiom* in economics. The work of PAUL SAMUELSON turns out to be crucial, yet not solely responsible for that. To shed light on this status change, this section is structured in three main parts. The first part 4.1 identifies the key drivers in the incorporation of physics methodology into economics. The second part 4.3 analyses briefly idea of natural sciences methodology as the only *right* form of methodology for any science, such as economics and analyses this methodological spillover. The third part 4.4 uses the tools given by rhetorical analysis to analyse ergodic economics.

4.1 From Boltzmann/Gibbs via Wilson to Samuelson

A first hint of who and what inspired much of the mathematisation of economics methodology from the 1930s onwards is given on the very first page of SAMUELSONS seminal *Foundations of Economic Analysis*. There SAMUELSON quotes the physicist and one of the founders of statistical

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[49] See MERTON et al. (1984) for an extensive enquiry into (un)attested origins of the first three quotes.

mechanics J. Willard Gibbs directly below the booktitle with 'Mathematics is a Language'. This quote alone has the power to explain most of the following.

This section answers the following question: What role did the ergodic hypothesis play in the mathematisation of economics? To answer this question the following quote from Samuelson (1968, pp. 11) stands out from the literature and sets us on the right track:

> [...] interesting [...] assumption implicit and explicit in the classical mind. It was a belief in unique long-run equilibrium independent of initial conditions. I shall call it the '**ergodic hypothesis**' by analogy to the use of this term in statistical mechanics. [...] Now, Paul Samuelson, aged 20 [...] as an equilibrium theorist he naturally tended to think of models in which things settle down to a unique position independently of initial conditions. Technically speaking, we theorists hoped not to introduce hysteresis phenomena into our model, as the Bible does when it says 'We pass this way only once' and, in so saying, takes the subject **out of the realm of science into the realm of genuine history**.
> [...] we envisaged an oversimplified model with the following **ergodic property**: no matter how we start [...] after a sufficiently long time it will become [...] a unique **ergodic state**. (bold emphasis added by MK)

This quote is telling in several ways, but confronts this analysis with the following questions:

1. How is it that Samuelson was fully aware of the underlying assumption of ergodicity to mathematical economics?[50]
2. Is there a superior methodology of economics? What can be said about that methodology compared to those of history and science?
3. Is the ergodic hypothesis empirically validated to rightly assume it in economics?

I tackle question 1 here and question 2 in 4.3.[51]

Question 1 Samuelson (1998, p. 1376) 'was vaccinated early to understand that economics and physics could share the same formal mathematical theorems', as he describes in 'How *Foundations* came to be'. Most likely because being the only (self-declared) disciple of Edwin Bidwell Wilson. Samuelson was a Ph.D. student at Harvard from 1936 to 1941 and from 1937-1940

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[50] Because Samuelson had such a big stake in moulding the type of mathematical economics, some authors labeled this line of research *Samuelsonian Economics*, (McCloskey, 2002; Weintraub, 1991).

[51] To answer question 3 a parallel paper is currently under construction. It will analyse several ergodic proofs from the 1930s to today in relation to their empirical verifiability.

a junior fellow.[52] E. B. Wilson was a versatile mathematician[53], who taught mathematical economics and statistics to Harvard graduate students from 1922-1945.[54] Importantly, Wilson was then again a student of J. Willard Gibbs, one of the founders of statistical mechanics.[55] Therefore, Samuelson (1998, p. 1376) sees himself as 'perhaps his [E. B. Wilson's] only disciple' and elsewhere '[E. B. Wilson] had been the only protegé at Yale of Willard Gibbs. Since I was Wilson's main protegé, that makes me kind of a grandson to Gibbs'[56].[57] Now the intellectual connection from Gibbs to Samuelson via Wilson is clearly visible.

Weintraub (1991) provides many hints on topics from mathematics and the natural sciences of interest to economists, many of them were brought to Samuelson's attention by Wilson, among others Le Chatelier's principle (p. 46), the relation between Lotka's *Elements of Physical Biology* and Wilhelm Ostwald's *Energetics*[58] (p. 47) and the work of George David Birkhoff on the mathematics of dynamical systems (pp. 49-53, 64, 70). The link to Birkhoff is important for this study, because he was one of the early founders and contributors to ergodic theory, delivering a first[59] ergodic theorem.

In the chapter *E. B. Wilson and the Gibbs tradition* Weintraub (1991, pp. 57-62) reconstructs, how the work of Samuelson can be seen as the continuation of a Gibbsian tradition in other scientific disciplines. Weintraub (1991, p. 61) summarizes the essence of the Gibbsian tradition as 'the prejudices of E. B. Wilson, a world-view shaped as a student of Willard Gibbs, were congenial to Samuelson's own program of making economics scientific by presenting the essential propositions of the subject in a mathematical, and thus clearly analyzable, form.' With this remarks, it becomes clear, why Samuelson was inclined to choose the Gibbs[60] quote right below the title in his *Foundations*.

The inheritance line regarding ideas, that are central for becoming acquainted with the ergodic hypothesis, thus reads Gibbs-Wilson-Samuelson. In order to cover more scientists relevant with

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[52] Harvard Junior Fellows in these days were engaging in a 'Faustian bargain [. . .] free to work on whatever they liked, but forbidden for three years to work toward any degree or Ph.D. dissertation.' (Samuelson, 1998, p. 1377)

[53] As an indication of Wilson's widespread scholarship: he served as the president of the American Academy of Arts and Sciences from 1927-1931 and he was the first 'managing editor of the Proceedings [of the National Academy of Sciences of the United States of America] for fifty years, from its first issue, dated January 15, 1915, until his death in December 1964' (Hunsaker and Mac Lane, 1973, p. 300). See also Bogorad (1995).

[54] James Tobin too reports of the influence Wilson's courses at Harvard's Public Health School had on him (Colander, 2007, p. 394).

[55] Gibbs, 1902; Lebowitz and Penrose, 1973.

[56] Barnett, 2007, p. 160.

[57] For self documented influence on him, see also Samuelson (1983).

[58] See Ostwald (1909) *Energetische Grundlagen der Kulturwissenschaft*. Ostwald is of further interest as a promoter of Gibbs' ideas and translater of Gibbs (1892) into German (Körber, 1961, p. XV, pp. 87–104).

[59] At least if one takes the publication dates, Birkhoff's proof was first, but it is conjectured, that Birkhoff has read a preprint of the proof that later became published as von Neumann (1932). G. D. Birkhoff then wrote his 1931 article already as a reaction to von Neumann (1932). Finally, von Neumann (1932) got published in the subsequent Proceedings of the National Academy of Sciences of the United States of America (PNAS) issue to Birkhoff (1931). For a detailed historical analysis see Zund (2002).

[60] Gibbs was also one of Irving Fisher's Ph.D. thesis supervisors at Yale University. Fisher is considered as one of the earliest American neoclassical economists.

respect to the ergodic hypothesis, it could be justly expanded to Poincaré-Gibbs-Birkhoff-Wilson-Samuelson, see figure 3. Surely it was through E. B. Wilson that Samuelson came in to close contact with the mathematics of a physics' image of the world in those times, including the meaning of the ergodic hypotheses. Wilson served as a window not only for Samuelson to interesting mathematical structures and was a source of inspiration. Furthermore, he was a rich source of concepts from the natural sciences waiting to be applied elsewhere and strongly supporting the use of mathematical concepts in other disciplines such as in economics.[61]

This historical remarks help to explain Paul Samuelson's special role and on how he came to know about ergodicity and it's fundamental relevance for the seeming success of (equilibrium) economics. In this regard, Samuelson's importance is hard to underestimate, that is why some scholars labeled this skein of economics as *Samuelsonian economics*[62].

Ludwig Boltzmann 1884, H-Theorem

Henri Poincaré recurrence theorem

J. Willard Gibbs (Yale 1871-1903) *Principles* 1902

George D. Birkhoff *PNAS* 1931
John von Neumann *PNAS* 1932

Edwin B. Wilson (Yale 1901-1911, Harvard PHS 1922-45,
PNAS Managing Editor 1915-1964)

Paul Samuelson (Harvard 1936-1941)
Foundations 1947 'Mathematics is a Language'
Economics 1948

Figure 3: Genealogy of the ergodic hypothesis in physics to the ergodic axiom in economics.

4.2 From rational mechanics to rational economics

The subtitle of the classic textbook on statistical mechanics Gibbs (1902) reads *Developed with Special Reference to the Rational Foundation of Thermodynamics.* What is meant by *rational* in this subtitle? It is science understood as an endeavour, which yields explanations via the use of reason and not metaphysical forces like ether, telepathy or ideological (especially religious)

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[61] A forerunner of this development took place around 1890s with the research programme of energetics by Ostwald, which tried to base everything including cultural phenomena on the principle of energy conservation (Körber, 1961, p. XV).

[62] McCloskey, 2002; Weintraub, 1991.

beliefs. In this way science is an attempt that strives for rationality and at the same time tries to work out to what extent observable natural phenomena can be explained by the means of rational explanations.[63] This approach to the understanding of nature became a desideratum of many researchers like Galileo Galilei, René Descartes and foremost Isaac Newton and big parts of the society, in order to free themselves from religious and/or royal dominion and tutelage. Nowadays they are conceived in one way or another as the founders of modern science beginning roughly with the Enlightenment period from 1650 onwards.

The French Revolution initiated later intellectual developments of the Enlightenment that led to the rise of positivism and later empiricism e.g. in the Vienna Circle and ultimately in an unconditional trust in science. As I write these lines, we live in a world that is dominated by the ideology of the superiority of technology and scientific argument in public discourse. There may be good reasons for that, not least if science is the worst form of understanding nature and society except all those other forms that have been tried from time to time[64]. In this sense, we nowadays experience a situation of great danger, since the most dangerous ideology could be the belief of being free of any ideology. Even if it is maybe the purest ideology of believing in (what one believes) science (is). It is in this sense calls for 'Disenlightenment' have to be understood.[65] They do not vote for the relegation of science but for an enlightened handling also and especially of its limitations even if they are much harder to see. Such conduct has generated Sorcerer's Apprentices of evolution and with it the spectre of a repairable world.[66]

Occasionally, science and scientist claim, they form their conclusion in a manner free of any ideology. Often, this false image is attributed (or even worse consciously used by) non-scientists to science, e.g. journalists or politicians[67]. Scientists have to fend this allegation off. Scientists have to make a stand against the utilization of their work by interests alien to science and make up their minds, whether it is possible to use research findings outside of their lab-like context e.g. in political discourse. This is not an easy thing to do, as it requires the scientist to be clear about his own ideological biases, prejudices and presuppositions. The Economist concludes '[Ms Reinhart and Mr. Rogoff] have sometimes been less careful in media articles. This is perhaps their biggest mistake. The relationship between debt and growth is a politically charged issue. It is in these areas that economists must keep the most rigorous standards.' This is so hard for us, because it deals with discipling not others but ourselves. This sitation resembles the debate that followed Friedman (1953) on the naturalistic fallacy of the character of economic science being solely positive or also normative.

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[63] Of course, this does in no sense imply, that science can answer everything or that its scope spans everything that is important.

[64] Paraphrasing Churchill's famous saying about democracy.

[65] Riedl (2004, p. 116) uses the German neologism 'Abklärung'.

[66] Riedl, 2004, p. 83.

[67] See the debate (Economist, 2013) on the existence of a certain harmful government debt threshold (Herndon et al., 2014; Reinhart and Rogoff, 2010), the great willingness of politicians to accept certain 'scientific' findings and what momentum this can create.

Coming to economics, Geisendorf (2009, 2010) point out wonderfully the strangeness of the absorption of the idea of rationality in economics. In economics, a dominant branch of rational expectations methodology developed, building the foundation of the current neoclassical mainstream, which usurped a specific understanding of rationality. Namely, not the discipline of economics being rational, in the sense that it uses reason in the form of logic, experiment, empirical evidence, measurability and replicability, but in defining the behaviour of elements of its subject area as being rational in the very limited sense of optimal. A criterion which is impossible to follow as a guidance in a world with uncertainty and an open future. All that just to justify the use of concepts like complete or full information, which are a trick to simplify the mathematics needed to solve these economic problems, similar to Boltzmann's trick of the ergodic hypothesis to simplify the mathematics involved in solving problems in statistical mechanics.

Although it has to be emphasised, there is nothing irrational in a scientific theory in which some or all of its actors behave boundedly rational when confronted with an uncertain open future. Hence, a scientific theory of boundedly rational (not omniscient) agents can be a rational theory.

4.3 Rhetoric of a Methodological Spillover from the Natural Sciences to Economics

This section adresses question 2: Is there a superior methodology of economics? What can be said about that methodology compared to those of history and science? More precisely, it targets two aspects. First, the methodological spillover of natural sciences methodology into economics, the so called *received view*. Second, the rhetoric of this spillover with respect to the crucial assumption of ergodicity to statistical mechanics.

Theory of Science in Economics – Logical Positivismus McCloskey (1983) forced economics and economists to start thinking about their rhetoric, '[e]conomics should become more self-conscious about their rhetoric [. . .]' (p. 482). Further, McCloskey (1983, p. 484) attests mainstream economics to follow a

> credo of Scientific Method, known mockingly among its many critics as the Received View, [that] is an amalgam of logical positivism, behaviorism, operationalism, and the hypothetico-deductive model of science. Its leading idea is that all sure knowledge is modeled on the early 20th century's understanding of certain pieces of 19th century physics.

A careful division of the relationship of logical positivism and the deductive-nomological (DN) model of science seems indicated.[68] McCloskey criticises the naïve believe in this amalgam as the exclusive scientific methodology of economics, that can be labeled *logical positivism* or *modernist methodology*. Logical positivism operates along the lines of empirically falsifiable experiences about worldly phenomena. Favouring reason and science over religion and authority started out during and after the age of Enlightenment from the work of Galilei, Descartes and Kant. In logical positivism the DN model is the preferred epistemology to draw conclusion from a priori laws in a deductive manner. The axiomatisation of mathematics culminated in the attempts of logicians and mathematicians[69] to base mathematics on, what was perceived as the purest form of reason namely, pure deductive logic by preserving consistency. Thereby following the socalled Hilbert programme[70]. Although prominently advertised in the 1950s by Friedman (1953), logical positivism was put aside by philosophy of science since the 1930s or 1940s.

Gödel or the mortal blow to logical positivism and Hilbert's Programme Philosophers close to or even members of the Vienna Circle pushed this movement further, among others Rudolf Carnap, Karl R. Popper, Moritz Schlick, and Ludwig Wittgenstein.[71] In the social sciences, logical positivism was incorporated by the founding father of sociology Auguste Comte[72] and across this way became the eminent epistemology for economics together with the attempt to its formalisation and axiomatisation. But through the workings of Gödel (1931) and his famous undecidability and incompleteness theorems this movement ended tragically. The only certainty left is the one about fundamental uncertainty.

Presenting Gödel and his undecidability theorems as witness to argue that all scientific inquiry, which follows a deductive approach like neoclassical economics, is eventually doomed to failure, is always an extreme position if not a thought-terminating cliché, because it argues with the ultimate consistency. Of course, this does not imply, that there aren't any statements, that can be proved or refuted. And to be distinct, there are provable statements and consistent theories, also about the economy. For economics this discussion in mathematics is relevant, whenever the attempt is undertaken to build large theories that try to retain consistency. The attempt to model an economy formally and deduce all statements from a priori laws and axioms seems thus at least to be prone to this undecidability issues. But half way to the ultimate there may well be decidable and (dis)provable statements, which go without contradiction with the rest of the

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[68] The DN model is used in Popper (1935) and is also known as the Hempel-Oppenheim model (Hempel and Oppenheim, 1948).

[69] Whitehead and Russell, 1910, 1927a,b.

[70] Hilbert, 1900.

[71] The Vienna circle pushed it so far, that some authors labeled this methodology as ultraempiricism.This particular and exclusive understanding of the substance of science fueled much of the debates in philosophy of science and scientific theory in the 20th century.

[72] In connection to Comte this epistemology is often called logical empiricism.

theory.

Methodological Remarks by von Hayek

Von Hayek (1994, p. 58) sees the natural sciences prevailing over the social sciences in dressing their findings in simple formulae, because the natural sciences define their research subject via the feasability of dressing it in simple mathematical formula,

> [t]here is, however, no justification for the belief that it must always be possible to discover such simple regularities and that physics is more advanced because it has succeeded in doing this while other sciences have not yet done so. It is rather the other way round: physics has succeeded because it deals with phenomena which, in our sense, are simple.

In this sense, everything that is describable in simple mathematical terms is physics, everything else is not. This is a recursive definition of the subject of a field, contingent on its successful application. Blaming the other disciplines, why they not succeed in finding simple mathematical regularities is actually not reasonable, if one follows von Hayek's reasoning. Such a statement is missing the point of the necessity of pluralism in the methodological choice, if the research subject requires it. This view explains why the mind and consciousness are not seen as a physics problem, because there are no convincing ways to describe the mind and consciousness in a mathematical formal statement (yet). Whereas, the pendulum is simple enough for human brains, to easily find mathematical expressions of its behaviour, therefore the pendulum is seen as a physics problem. Additionally, von Hayek ([1974] 1989, p. 3) points out, that the success in physics rests on the stable existence of oberservables, whereas in economics and social sciences a variable enters economic theories and often becomes treated as if it were an observable, 'which happens to be accessible to measurement.' This again is a case, where a method is used reversely, but the reverse must not always be true as well, as a reversed logical implication doesn't work in opposite direction with turned direction of causality. For example, the mass of a solid body is an oberservable, and so mass is a potential element for physical theories. The German stock index (DAX), Dow Jones Industrial Average or any other stock index is not an observable in the original sense, because the DAX on January, 1^{st} in 1988 consists not only of different stocks than the DAX on January, 1^{st} in 2015, most of the companies of these days don't even exist anymore. Furthermore, the meaning the whole system, any central bank or an arbitrary individual attaches to the DAX is likely to have completely changed in totally different ways during this time and is therefore likely to be different at arbitrary moments in time. The same holds for gross domestic product (GDP), unemployment rate, money aggregates, consumer price index (CPI) and many if not most economic variables. All these quantities are human artefacts. Humans attach meaning to it for pragmatic reason and these reasons can change (for legitimate causes). For example, how to quantify what is understood at a time as a useful measure of unemployment changed tremendously in the $20th$ century, owing a lot to economic debates regarding structural/voluntary unemployment following different ideas or theories of different economic thinkers like Marx, Keynes or neoclassical conceptions. This makes obvious, how problematic it is, to interpret a time series of say unemployment rates or CPI.

Von Hayek is also insightful in reflecting on how a virtue may become a vice in thinking beyond the Popperian falsification corset, if the scientist is confronted with a complex phenomena:

> The advance of science will thus have to proceed in two different directions: while it is certainly desirable to make our theories as falsifiable as possible, we must also push forward into fields where, as we advance, the degree of falsifiability necessarily decreases. This is the price we have to pay for an advance into the field of complex phenomena.[73]

4.4 The Rhetoric of Ergodic Economics

As is shown in chapter 3, (non-)ergodicity as a model property is of fundamental importance for economic theorizing, because it led virtually to the elimination of time from economics as a creative force. It is than important, if this elimination is hardly ever made explicit in the literature.

McCloskey (1983, p. 507) elucidates, what can be expected of a careful investigation into rhetoric: 'unexamined metaphor is a substitute for thinking – which is a recommendation to examine the metaphors, not to attempt the impossible by banishing them'. Hence, in this section a special rhetoric in economics is carved out, regarding the implicit assumption of ergodicity, guiding most of $20th$ century reasoning in economics like almost no other model property.

> A rhetoric of economics makes plain what most economists know anyway about the richness and complexity of economic argument but will not state openly and will not examine explicitly.
> The invitation to rhetoric, however, is not an invitation to irrationality in argument. Quite the contrary. It is an invitation to leave the irrationality of an artificially narrowed range of arguments and to move to the rationality of arguing like human beings.[74]

Quite the contrary of what McCloskey decribes in the above quote is the case for ergodicity in economics. After making the experience of my own instruction in economics and statistics, after having done a careful literature review and after having talked to many scholars at conferences during and after talks of mine on this topic, it is not at all obvious, that many economists know, that they are implicitly assuming ergodicity in their models, although it is necessary for their models to possess a (unique equilibrium) solution. Therefore, this chapter analyzes what

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[73] Von Hayek, 1994, p. 58.

[74] McCloskey, 1983, p. 509.

contributed to the current situation of what could be called a predominance of ergodic economic models.

The key assumptions of mainstream neoclassical economics, that are stated explicitly over and over again also by the many critics, read as follows:

- use of the representative agent (firm/household/government),
- rational expectations,
- efficient markets.

Ergodicity is clearly missing on this list. Therefore, the rhetoric of ergodic economics consists of concealing this fundamental assumption. That is why I am devoted to this topic. Metaphorical loaning from physics was not always and not in every detail concealed, 'the core of neoclassical research program is a mathematical metaphor approximated from physics in the 1870s which equates potential energy to "utility", forces to "prices", commodities to spatial coordinates, and kinetic energy to the budget constraint. The early generations of neoclassical economists constructed their system from such metaphors, and openly acknowledged them.' (MIROWSKI, 1989, p. 176) This is clearly not the case for the assumption of ergodicity. Only in more recent epistemological analyses of economics, ergodicity is recognised as central for neoclassical and todays mainstream economics.[75] For HIRTE and THIEME (2013, p. 71) ergodicity even counts to the 'Grundaxiome'.

MCCLOSKEY (1983, pp. 500) finds several mechanisms of sociological influence in the process of scientific discovery[76], among others appeal to authority, appeal to relaxation of assumptions, appeal to hypothetical toy economies and appeal to analogy. This is not the first time to recall the enormous amount of influence the impressive œuvre of PAUL SAMUELSON had. A considerable amount of this influence appears to be by appeal to authority. The reason for this appeal can be seen in the 'air of easy mathematical mastery'[77], which was a long-cherished desideratum at a time where all the (social) sciences were envying the queen of the sciences, viz highly mathematised physics.[78]

How else then as faithful believe in the master of (at his time) modern economics can it be explained, that there is almost no reference stating explicitly the importance of the ergodic hypothesis for economics except the one from SAMUELSON (1968) himself, who has been '[t]hroughout his career, [...] the master of scientific rhetoric, continuously and consciously hinting at parallels between neoclassical theory and twentieth century physics, and just as consciously denying them, usually in the very same article.' (MIROWSKI, 1989, p. 186) Any of the above mentioned findings of a process of mathematisation get amplified, if the underlying mathematical field is not yet fully

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[75] See HEISE, 2012, p. 85 and HIRTE and THIEME, 2013, pp. 28, 64.

[76] Which developed nowadays into a research area of its own called sociology of scientific knowledge (SSK).

[77] MCCLOSKEY, 1983, p. 500.

[78] See also MIROWSKI, 1989, on the misuse of physics metaphors in economics.

systematized and its ramifications not entirely understood. This is the case for ergodic theory as well as for example for KAM Theory, of which it is an important part of.[79]

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[79] Dumas, 2014.

> [O]nce Pandora's box was opened, there seemed to be no stopping the spread of probabilistic concepts in physics. Probabilistic gas theory led to statistical mechanics, which begat quantum mechanics, which begat probability waves, which begat nonergodic and weakly stable systems [. . .] all of which begat a seemingly accidental universe.[80]
>
> Philip Mirowski

5 Conclusion

To understand the process of formalisation and axiomatisation in economics post WWII the relationship between the work of Paul Samuelson and Edwin Bidwell Wilson turned out to be crucial. The evidence of Wilson's influence on Samuelson was mentioned by Samuelson himself many times, besides worked out by several historians of economics and economists close to him such as von Weizsäcker (1997, p. 92) 'Samuelson ist ganz wesentlich verantwortlich für diese methodische Wende im Fach. Er wurde in seinen jungen Jahren von bedeutenden Physikern beeinflußt und hat sich zum Beispiel in seinen *Foundations* die präzise Begrifflichkeit der Physik zum Vorbild genommen.' Apart from this and the historical fact that Samuelsonian economics became mainstream, there is much critical literature on the methodological and metaphorical loaning[81].

The direction of the mathematisation of economics after WWII[82] is unthinkable without the tacit assumption of ergodicity underlying almost all parts of standard textbook economics. The realisation of the relevance of non-ergodicity as the most important model property of a form of economics, that includes the explanation of novelty and evolutionary change, challenges the line of thought, which mathematical economics follows since the 1940s. One reason for the adoption of this line of thought is the fact, that economic methodology draw on classical and later statistical mechanics, at which core lies the use of ensembles in Gibbsian statistical mechanics. In the introduction to his *Cybernetics* Wiener ([1948] 1985, p. 4) reminds us, that cybernetic ideas 'were all very much in the spirit of the thought of the time, and I do not for a moment wish to claim anything like the sole responsibility for their introduction.' What is true for the advent of cybernetics is also valid for the use of methods from the natural sciences (especially physics) in economics. The demand to put economics on more scientific footing lay in the spirit of the thought of the time. Still, Samuelson did ingeniously excel in this movement with his seminal dissertation *Foundations of Economic Analysis* and soon after it with his famous textbook

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[80] Mirowski, 1991, p. 64.

[81] First and foremost see the work of Philip Mirowski.

[82] As decribed among others in Weintraub (2002).

Economics. Nevertheless, Samuelson (1968) gives the impression being well aware of the crucial assumption of ergodicity underlying the whole movement of the increasing mathematisation of economics, from today's point of view, he may well have underestimated its criticality to the whole.[83] Regrettably, this knowledge got lost through the years, despite its paramount importance, to large part because it was rarely mentioned at all, neither thoroughly brought into light. This is what this piece aimed at.

In a parallel world, in which the ergodic hypothesis obtained proper attention in economics, it is conceivable, the process of mathematisation taking a different road. One, which incorporated 'the new insights of thermodynamics or the theory of relativity or quantum mechanics' (Mirowski, 1989, p. 178) and advanced earlier and deeper into topics like path dependence, which struck the world of academic economics only in the late 1980s[84] or the use of simulations to explore not only different contingent worlds, but also possible worlds in which time had chosen another trajectory.

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[83] Or as Samuelson (1952) suggests, he is simply a honest believer.

[84] Arthur, 1989; Arthur et al., 1987; David, 1985.

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