The Backbending Supply Curve of Labor: Comment on Buchanan; with his Reply

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In a recent issue of this journal, Professor Buchanan has used the case of the backbending supply curve for labor to raise the question whether the current choice-theoretical "orthodoxy" does not, at least in this instance, represent "doctrinal retrogression." Buchanan notes (p. 385) that "the phenomenon is almost always referred to as 'peculiar' or 'perverse.' " He points out that if the explanation of the backbending supply curve case is conducted so as to keep in clear focus "the fundamental reciprocity of demand and supply"—a pre-Hicksian concept—any impression that there is something "strange or bizarre" about the case is dispelled entirely. "No resort to a Giffen-like paradox is required" (p. 384). This pedagogic point is well taken and illustrated below as a preliminary to a critique of Buchanan's main argument.

Professor Buchanan goes on to argue that the current (Hicksian) "orthodoxy" befuddles the issue in always explaining the backbending segment of the labor supply curve by invoking a negative income-effect (from a wage rise) of sufficient magnitude to offset the positive substitution effect. This explanation provides sufficient conditions but,

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2. Buchanan attributes the formal development of the concept to Walras. It is usually attributed to J. S. Mill. Cf. Principles of Political Economy, bk 3, chap. 18, sec. 4, where the concept is so named, and the first of Mill's earlier Essays on Some Unsettled Questions of Political Economy, where the same analysis is carried through but the term not coined.
3. Buchanan, p. 385. On the following page, Buchanan indicates his preference for Lionel Robbins' "much more straightforward" analysis. In his "On the Elasticity of Demand for Income in Terms of Effort," Economica 10 (June 1930), Robbins relied directly on inelasticity of demand for income in terms of effort in explaining the backbending supply case. The issue is whether this inelasticity can, in its turn, be derived without invoking a Hicksian income-effect as above.
he maintains, fails to elucidate the necessary conditions for this case to arise. In support of this contention, Buchanan produces a diagram which he interprets as illustrating the possibility that, along an income-compensated demand-for-income schedule, the reciprocal "voluntary" supply of labor may decrease as the wage is increased. We must allow the possibility that this income-compensated supply curve may bend backward, Buchanan argues, because we cannot a priori impose the restriction that "all fully compensated demand curves exhibit an elasticity greater than unity in absolute value throughout their whole range."  

Consider now Fig. 1. On the horizontal axis, OD is the length of the "day," labor supplied being measured leftwards from D. The vertical axis measures Robbins income. The individual's endowment is represented by the point E₀—he has DE₀ of property income. E₀R is the standard offer curve, or endowment-constant reciprocal demand curve, as used in the international trade literature, by Walras and Marshall, and by Robbins—the pre-Hicksian references given by Buchanan for the reciprocal demand concept. No two points on E₀R have the same Hicksian income, since the individual's gains from trade increase monotonically with the distance from E₀. Machlup's "good undergraduate exercise," mentioned by Buchanan, consists in showing that at point X, where E₀R has a vertical tangent, the price elasticity of labor supply is zero and the elasticity of demand for Robbins income is consequently unity. Above X, income demand is inelastic, and this is associated with backward bending labor supply—but because the Hicksian income-effect swamps the substitution effect. Suggestions

4. The infelicity of "income-compensated demand for income" arises, of course, from the simultaneous use of two distinct income concepts, namely, Robbins's (which stands for an index of the quantities of all non-leisure goods commanded) and Hicks's (an index of utility). For the purpose of this note, Buchanan's double-barreled usage is retained.

5. Buchanan, p. 387, italics added. It is true that we cannot assert this restrictive condition for all income-compensated demand curves. Such a curve drawn for one good in a world of n goods need not have the property in question. In the case under discussion, however, "income" stands for all goods but one. In a two-good world, simply postulating continuously diminishing marginal rate of substitution is sufficient to ensure that total expenditures in terms of one good in exchange for the other are increased as the exchange value of the latter good is lowered along its income-compensated demand schedule. Note, below, Buchanan's distinction between "total" and "voluntary" expenditures.
that there is anything "peculiar" about this range of $E_0R$ are clearly out of place. A "paradox" would be present only if we could have a case such as that indicated by the broken-line segment $ZR'$ on $E_0R'$. To the right of point $Z$ (at which the labor-supply elasticity is $-1$), the basket of all non-leisure goods would have to be Giffen—the usual reason why we are content to rule out negative labor-supply elasticities larger than 1 in absolute value. All this is in accord with Buchanan's introductory comments.

In Fig. 2, we consider Buchanan's counterexample. The indifference curve labeled $U_0$ represents the income-compensated demand curve
that we are concerned with. With this construction, the endowment should be removed from the *ceteris paribus* conditions of the problem and be replaced with some historical solution as the fixed point of reference. Let this fixed point be at $A$, where the price line through $E_0$ is tangent to $U_0$, so that $A$ is the only point that the $U_0$ locus has in common with $E_0R$ of Fig. 1. All other points on $U_0$ lie off $E_0R$.

Buchanan's conceptual experiment then goes as follows. The individual's initial situation is at $E_0$, facing the wage line labeled $a$, under which conditions he will choose point $A$. $B$ is a point indifferent to $A$ and sustained by the wage line $\beta$—a higher real wage than $a$. Buchanan’s compensation scheme for inducing the individual to choose $B$ involves “‘taxing’” him $E_0E'$ of labor without pay but giving him the opportunity to sell additional labor at the wage rate $\beta$. Choosing $B$, the individual is, of course, expending a higher total amount of labor (equal to $B'E_0$), but only $B'E'$ of this total is “‘voluntarily’” supplied. The amount $B'E'$ is less than $A'E_0$, so, Buchanan concludes, we have an instance of the “‘voluntary’” supply of labor decreasing with an increase in the wage rate *although* the influence of income-effects has been eliminated from the analysis.

This paradoxical conclusion tends to entice the reader into sundry philosophical reflections on Buchanan’s distinction between “‘voluntary’” and “‘total’” supply, its analytical usefulness or lack of it, etc. Semantic argument on this issue is likely to prove inconclusive.7 We will do better simply to consider the consequences of adopting the analysis of Buchanan. The following considerations seem germane:

1. It is irrelevant to Buchanan’s experiment whether the initial point $A$ (in Fig. 2) is considered to lie above or below $X$ on $E_0R$ (in Fig. 1). Suppose, in the analysis above, we started out with $A$ below $X$. Then, the endowment-constant supply curve is upward sloping while Buchanan’s “‘voluntary’” supply bends back. We could just as easily obtain a case with $A$ above $X$ where the standard curve bends back while Buchanan’s goes the other way. Hence, Buchanan’s analysis does not simply complement the sufficient conditions of the usual

6. Cf. n. 5 above.

7. For a fuller account of Buchanan’s position on these matters, cf. his *Cost and Choice* (Chicago, 1970). I should perhaps confess that I am dodging the deeper issues mentioned above, in part because of my general agreement with Buchanan’s position in this recent book.
analysis by pointing out an overlooked possibility; instead, it introduces an entirely new framework yielding results which, qualitatively, agree with standard analysis only by chance.

2. Buchanan claims that his conclusions do not depend on the compensation-scheme used in his particular illustration. This, unfortunately, is simply false. To see this, take the same initial situation, but assume that the compensation is taken, not in forced labor, but by expropriating $E''E_0$ of the individual's property income. With his endowment at $E''$ and facing the $\beta$ wage rate, the individual will again go to $B$, supplying a total $DB''$ of labor—but all of it "voluntary" this time. Obviously, if compensation is taken in terms of Robbins income, "voluntary" labor supply will invariably be positively related to the wage rate along any income-compensated curve.

3. With the ordinary backward bending curve, the experiments will be reversible. Labor supplied will increase if the wage is lowered. Buchanan's analysis is peculiar in that, whereas he obtains a decrease in "voluntary" labor supply from point $A$ when the wage rate is raised, he cannot get an increased supply by lowering the wage from the same initial position. Any initial position with all of the labor supply "voluntary" will create the same difficulty, since compensation cannot be given by lengthening the "day."

4. Finally, consider compensation schemes—entirely permissible—that take more forced labor than the minimum necessary but compensate for the "unnecessary" labor taken with additional Robbins income. To go from $A$ to $B$, we could in this manner select a compensation scheme putting the individual anywhere on $E'B$. By selecting a point arbitrarily close to the income-compensated schedule, it is always possible to produce the conclusion that "voluntary" labor supply decreases with an increase in wages. We have already seen that the opposite conclusion can always be reached by choosing a point arbitrarily close to $E''$.

Two conclusions suggest themselves. (i) Writers who, using the standard analysis, suggest that there is something "peculiar or perverse" about the backbending supply of labor case are simply sloppy

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9. We are then refusing to face the conundrums that arise from contemplating points above $B$ on the same wage line.
users of the Hicksian instrument, not innocent victims of defects inherent in it. (ii) The alternative apparatus suggested by Buchanan should not be adopted. Since the conclusions it generates are essentially arbitrary, nothing can be learned from its use. 10 Economics is anything but proof against doctrinal retrogression—but we do not here have a case of it. 11

REPLY BY PROFESSOR BUCHANAN

The main point of my paper was that a backbending supply curve for labor is fully consistent with a theory of demand that is, itself,

10. The danger is that something may be lost. Buchanan’s concluding paragraph states inter alia: “In the relevant exchange under consideration, the individual acts as a supplier of leisure not as a demander. . . . The behavior that is observed as we confront the individual with alternative wage bids cannot be treated both as a supply and as a demand function for leisure. The behavior can be treated as either a supply function for leisure (labor) or a demand function for income. . . .” Although I am convinced that Buchanan intends nothing of the sort, the first two sentences quoted read like a rejection of Wicksteed’s concept of (a supplier’s) “reservation demand”, just as the “either . . . or” of the last sentence seemingly renounces the “fundamental reciprocity” which Buchanan began by emphasizing.

11. The case considered here has two special features in that the system has only two goods and that one of them is the “day,” the 24 hours of which the decision maker has to endure, “voluntarily” or not. Our negative conclusions, therefore, do not carry over to the use of income-compensated constructions in general. Some reflections on that topic, prompted by Buchanan’s paper, may nonetheless be in order: (i) I suspect we would do well to file away and forget the Hicksian-income-compensated demand schedule. Utility-constant schedules have the gains from trade invariant and, at worst, rule them out of existence—a curious notion for economists to entertain. A good problem in economics requiring this kind of (empirically inoperational) “welfare invariance” is hard to imagine; it seems almost a contradiction in terms. (ii) Production-possibility-constant demand schedules, usually constructed with compensation schemes having a zero net value in terms of Robbins income when evaluated at terminal prices, will have their uses. Cf. Buchanan, “Backbending Supply Curve,” p. 387, n. 9. But then it should be agreed, as apparently it is not at present, that such constructions do not reflect “only substitution effects.” Cf. Leijonhufvud, On Keynesian Economics and the Economics of Keynes (New York, 1968), pp. 259–71, esp. p. 266, n. 6. (iii) To be safe, the use of these constructions had better be confined to topics where the particulars of the compensation scheme assumed either are dictated directly by the problem or constitute the problem. All three of these tentatively suggested “prescriptions” seem to me applicable to that large class of traditional problems of international trade or public finance where the state withdraws resources from the private sector via a specific activity and returns their equivalent in general purchasing power or, alternatively, withdraws general purchasing power to subsidize a specific activity.
consistent in a full general-equilibrium setting. The backbending segment of a labor supply curve is nothing more than the inelastic portion of the reciprocal demand curve that mirrors the same behavior. It is not at all similar to something like the Giffen paradox in upsloping demand curves, which the standard textbooks suggest. Professor Leijonhufvud apparently accepts this basic point. His criticism reduces, therefore, to the more technical details of the compensation apparatus which I introduced by way of demonstrating the basic argument.

I recognize the ambiguities that are almost necessarily present in any attempt to introduce precise compensating variations that are objectively measurable, meaningful at the level of individual response, and consistent with overall resource fixity. The apparatus that I used is vulnerable on this score, as Professor Leijonhufvud’s note indicates. I should not, however, accept his claim that my conclusions depend on the particulars of the compensation scheme. With reference to his Fig. 2, if the compensating tax is measured in the vertical dimension and is in the amount $E_0E''$, Leijonhufvud claims that my conclusions do not hold and that there is more rather than less labor voluntarily forthcoming at the higher wage. But surely this is a rather tenuous claim, since an amount $E_0E'$ of this labor is required to pay the compensating tax and can scarcely be said to be offered wholly in response to the wage available in the market.

I should not, however, becloud the central issues between us by discussion of geometrical-analytical detail. My emphasis was upon the development of an analytical framework that helps us to explain the workings of the whole market system and not its component parts. The central question is whether or not the conceptual separation of income and substitution effects was or was not doctrinal retrogression, as I had suggested. On this we shall continue to disagree. Something more than "sloppy use" is indicated when sophisticated and intelligent modern economists are misled, along with their students, on something as elementary as the supply curve for labor. I do not argue for the use of any specific "alternative apparatus." I should argue strongly for clarity in the basic conceptual analysis. Axel Leijonhufvud can avoid intellectual error even when he uses tools that are fraught with confusion; unfortunately, many of his professional colleagues cannot.