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The Qualitative Expectations Hypothesis

Roman Frydman, Søren Johansen, Anders Rahbek, and Morten Nyboe Tabor

The Qualitative Expectations Hypothesis – I

The Qualitative Expectations Hypothesis (QEH) is a new approach to modeling macroeconomic and financial outcomes.

QEH recognizes that economists and market participants alike face **ambiguity** about which is the correct quantitative model of the process driving outcomes.

Building on Frank Knight's distinction between risk and "true uncertainty," QEH formalizes ambiguity by opening an economic model to **unforeseeable change**.

- The defining feature of unforeseeable change is that it cannot "*by any method be [represented* ex ante *] with an objective, quantitatively determined probability*" (Knight, 1921, p. 321).

The Qualitative Expectations Hypothesis – II

Opening a model to unforeseeable change, and yet aiming to confront the model's predictions with empirical evidence, poses considerable challenges:

1. The model's **quantitative predictions** are at best relevant for a limited period of time;
   - eventually any such prediction becomes inconsistent with time-series data.
   - in this sense, the model does not generate quantitative regularities of movements in the data over time.

2. For the model to generate even **qualitative regularities**;
   - it must replace probabilistic representations of change with formalizations that recognize that, as Karl Popper put it, the "*future is objectively open*."

3. **Rethinking econometric methodology**.
   - Requires an econometric approach that recognizes the existence of unforeseeable structural change (as David Hendry has emphasized).

The Qualitative Expectations Hypothesis – III

QEH proposes a way forward that might overcome these challenges.

- Regardless of whether QEH, or some other, yet-to-be invented approach, turns out to be useful in this regard, **recognizing the inherent limits to what we can know about the future** appears necessary for developing epistemologically coherent and empirically relevant macroeconomic and finance models.

- By design, recognizing unforeseeable change implies that we can only uncover **qualitative regularities** in time-series data.

The Consensus Conception of Economic Science

Today, economic models must account for **quantitative regularities** in time-series data to be considered scientific.

Both REH and behavioral-finance models adhere strictly to this consensus, although they differ in a number of important ways.

We illustrate how REH and behavioral-finance models follow this consensus in the context of a simple stock-price model.

This sets the stage for showing how QEH formalizes the ambiguity confronting economists and market participants alike about which is the correct model of the process driving outcomes.

A Simple Stock-Price Model

- The model rests on an assumption that market participants bid the price to the level that satisfies the following **no-arbitrage condition**:

  $$p_t = \gamma \left(\mathcal{F}_t \left(d_{t+1} \right) + \mathcal{F}_t \left(p_{t+1} \right) \right)$$

  where p_t is the market price, d_t are dividends, $\mathcal{F}_t \left(\cdot \right)$ stands in for the **market's forecasts**, and $0 < \gamma < 1$ is a discount factor.

- Dividends d_t depend on earnings x_t:

  $$d_t = b_t x_t + \varepsilon_t,$$

  where b_t is the **time-varying impact of earnings on dividends**.

- Earnings $x_t > 0$ follow a martingale process:

  $$E \left(x_t | x_{t-1} \right) = x_{t-1}.$$

A Complete Stochastic Process

To account for quantitative regularities in time-series data, REH *and* behavioral-finance models specify a **complete dynamic stochastic process** driving outcomes.

- Typically, the impact of earnings on dividends over time is constant, $b_t = b$, so that

$$d_t = bx_t + \varepsilon_t, \qquad E\left(x_t | x_{t-1}\right) = x_{t-1}.$$

- Alternatively, such models could also assume a stochastic process for b_t.

Implies that the model makes **quantitative predictions of future outcomes**.

- For example, the conditional expectation of d_{t+1}:

$$E\left(d_{t+1} | x_t\right) = E\left(bx_{t+1} + \varepsilon_{t+1} | x_t\right) = bx_t.$$

Rational Expectations Hypothesis (REH)

John Muth's **principle of coherent model-building**:

> *[Participants' expectations] are essentially the same as the predictions of the* relevant economic theory (Muth, 1961, p. 316, emphasis added).

Once an economist hypothesizes that the complete stochastic process

$$d_t = bx_t + \varepsilon_t, \qquad E\left(x_t | x_{t-1}\right) = x_{t-1},$$

characterizes how dividends *actually* unfold over time, relying on Muth's principle leads him to represent the market's forecast with REH:

- Conditional expectations serve as a representation of the market's forecasts which is **consistent with the quantitative predictions of the model**:

$$\mathcal{F}_t\left(d_{t+1}\right) = E\left(d_{t+1} | x_t\right) = bx_t.$$

This implies that the stock price equals the present value of future expected dividends:

$$p_t = \gamma\left(\mathcal{F}_t\left(d_{t+1}\right) + \mathcal{F}_t\left(p_{t+1}\right)\right) = \sum_{i=1}^{\infty} \gamma^i E\left(d_{t+i} | x_t\right).$$

Logical Implications of Assuming a Complete Stochastic Process – I

Irrationality of Diversity of Forecasting Strategies

Robert Lucas: $\mathcal{F}_t(d_{t+1}) = E(d_{t+1}|x_t) = bx_t$ is the only way to characterize rational forecasts.

- Any forecast $\tilde{\mathcal{F}}_t(d_{t+1})$ that differs from $E(d_{t+1}|x_t) = bx_t$ leads to systematic forecast errors.
- Reliance on non-REH representations presumes gross irrationality
- As Lucas put it, it is the “wrong theory” of quantitative regularities.

Only Risk

Lars Peter Hansen (2013): “*Only allows for risk as conditioned on the model.*”

- Risk arises from exogenous shocks that are fully specified probabilistically.
- No Knightian uncertainty.

Logical Implications of Assuming a Complete Stochastic Process – II

For illustration, assume that at time $T+1$ the coefficient b undergoes an unforeseeable change from b to B:

$$d_T = bx_T + \varepsilon_T, \qquad d_{T+1} = Bx_{T+1} + \varepsilon_{T+1}.$$

- $\mathcal{F}_T\left(d_{T+1}\right) = bx_T$ results in a forecast error:

$$\mathrm{err}_{T+1} = d_{T+1} - \mathcal{F}_T\left(d_{T+1}\right) = \left(B - b\right)x_{T+1} + b\Delta x_{T+1} + \varepsilon_{T+1}.$$

- The component $b\Delta x_{T+1} + \varepsilon_{T+1}$ is stochastic and represents **risk**.
- The component $\left(B - b\right)x_{T+1}$ represents **Knightian uncertainty** that arises from unforeseeable change.

Illustrates Knight's argument that standard probabilistic risk misses the "true uncertainty" that arises from unforeseeable change:

> *if all changes [...] could be foreseen for an indefinite period in advance of their occurrence, [...] profit or loss would not arise* (Knight 1921, p. 198).

Logical Implications of Assuming a Complete Stochastic Process – III

The stock-price, p_t, equals the present value of *expected* future dividends:

$$p_t = \sum_{i=1}^{\infty} \gamma^i E\left[d_{t+i} | x_t\right].$$

The stock price, p_t, can be rewritten as the present value of *actual* future dividends, p_t^F, plus a mean-zero forecast error, η_t:

$$p_t = p_t^F + \eta_t, \qquad \text{where } p_t^F = \sum_{i=1}^{\infty} \gamma^i d_{t+i} \text{ and } E\left(\eta_t | x_t\right) = 0.$$

Once an economist hypothesizes that his probabilistic specification of the dividend and price processes represent how these outcomes actually unfold over time, the **market delivers an allocation that is nearly as perfect as that of an omniscient planner**.

This yields the most far reaching implication of these models: **The Efficient Markets Hypothesis**.

The Efficient Market Hypothesis (EMH)

> *Unfettered markets populated by "rational" participants deliver a nearly perfect allocation of resources.*

The common interpretation of EMH as the statement that "*In an efficient market, prices 'fully reflect' available information*" (Fama, 1976, p. 133).

Misses the key point:

- **EMH is an artifact of the assumption of no unforeseeable change.**

As we shall point out later, once we open the model to such change, EMH does not follow:

- Even if information is *not* asymmetric, that is, it is completely available to every market participant.

Shiller's Excess Volatility Puzzle

Robert Shiller (1981):

- Any REH model predicts that the stock price

$$p_t = \sum_{i=1}^{\infty} \gamma^i E\left[d_{t+i} | x_t\right]$$

  should fluctuate less than the perfect foresight price

$$p_t^F = \sum_{i=1}^{\infty} \gamma^i d_{t+i}$$

- Shiller found the opposite empirically: "*stock prices fluctuate too much to be justified by subsequent changes in dividends.*"

The Behavioral-finance Approach

Behavioral-finance economists have:

- Highlighted that REH models assume away the role of psychological factors in driving outcomes.
- Persuasively demonstrated the empirical relevance of these factors.
- Argued that these factors might contribute to REH models empirical difficulties.

Following the disciplinary consensus, behavioral-finance models specify a **complete probability distribution** of outcomes (as REH).

- This has led behavioral-finance theorists to embrace the belief that REH represents how rational individuals forecast.

Gross Irrationality of Behavioral-Finance Representations

In contrast to their REH counterparts,

- Behavioral-finance models assume that the market's forecasts of outcomes are driven by **psychological factors**.
- Hence, they *must* represent the market's forecast as inconsistent with the model's formalization of how outcomes actually unfold over time.
- Consequently, Lucas's argument applies:
    - behavioral-finance models presume that market participants are **grossly irrational** in the sense that they ignore systematic, observable forecast errors in perpetuity.

Rationality Under Imperfect Knowledge

John Maynard Keynes understood early on that when knowledge is imperfect, *rational* decision-making relies on both fundamental and non-fundamental factors, such as psychological considerations and social conventions:

> *We are merely reminding ourselves that [...] our* rational *selves [are] choosing between alternatives as best as we are able, calculating where we can [on the basis of fundamentals], but often falling back for our motive on whim or sentiment or chance* (Keynes, 1936, p. 163, emphasis added).

The QEH Stock-Price Model

The First Component of QEH

*By opening a model to **unforeseeable change**, a QEH model recognizes **ambiguity** about which is the correct quantitative model of the process driving outcomes.*

The defining feature of unforeseeable change is that it cannot "*by any method be [represented* ex ante*] with an objective, quantitatively determined probability*" (Knight, 1921, p. 321).

Opening the Model to Unforeseeable Change

As before, consider

$$d_t = b_t x_t + \varepsilon_t.$$

We open the model to **unforeseeable change** as follows:

1. Impact of earnings x_t on dividends d_t is positive at all times: $b_t > 0$.
2. Periods of time where the unforeseeable change in b_t is "moderate":

$$\text{Moderate Change (MC):} \quad \frac{|\Delta b_{t+1}|}{b_t} \leq \frac{|\Delta x_{t+1}|}{x_{t+1}}.$$

   MC implies the **qualitative regularity** of positive co-movement: $\Delta d_t \Delta x_t \geq 0$ (up to an error term). That is, $\Delta x_t > 0$ (< 0) implies $\Delta d_t > 0$ (< 0).

This implies that there are periods of time where b_{t+1} lies within the interval:

$$b_{t+1} \in \mathcal{I}^b_{t+1} = \left[b_t \left(1 - \frac{|\Delta x_{t+1}|}{x_{t+1}} \right)^+, b_t \left(1 + \frac{|\Delta x_{t+1}|}{x_{t+1}} \right) \right].$$

Note: As the change in b_t is unforeseeable, we do not specify a mechanism determining the value of b_{t+1} within the interval $\mathcal{I}^b_{t+1}$.

Unforeseeable Change Implies Qualitative Predictions

Allowing for unforeseeable change in b_t recognizes the **ambiguity** faced by the economist about the process driving dividends:

- As the value of b_{t+1} within the interval $\mathcal{I}^b_{t+1}$ is not known at time t, there is ambiguity about the quantitative model for the dividend process at time $t+1$.

Consequently, the QEH model does not generate quantitative predictions of future outcomes.

- For example, we cannot compute conditional expectations of future outcomes.

Instead, the QEH model makes **qualitative predictions** about future outcomes. We formalize these qualitative predictions with the Qualitative Expectations.

The Qualitative Predictions of the Model

We define the **Qualitative Expectation (QE)** of the stochastic interval $\mathcal{I}$ as:

$$QE_t(\mathcal{I}) = [E(X_L|x_t), E(X_U|x_t)], \quad \text{where } \mathcal{I} = [X_L, X_U]$$

i.e. $QE(\mathcal{I})$ is the conditional expectation of the bounds of the interval.

- The hypothesized unfolding of b_t implies that the value of dividends, d_{t+1} lies within the interval:

$$\begin{aligned} b_{t+1} &\in \mathcal{I}^b_{t+1} = \left[b_t \left(1 - \frac{|\Delta x_{t+1}|}{x_{t+1}} \right)^+, b_t \left(1 + \frac{|\Delta x_{t+1}|}{x_{t+1}} \right) \right]. \\ d_{t+1} &\in \mathcal{I}^d_{t+1} = \mathcal{I}^b_{t+1} x_{t+1} + \varepsilon_t. \end{aligned}$$

- We use the QE to derive the expected intervals for dividends d_{t+1}:

$$QE_t\left(\mathcal{I}^d_{t+1}\right) = b_t x_t [L, U],$$

  where the bounds L and U depend on the model for x_t.

The **qualitative prediction** of the QEH model is that d_{t+1} is expected to lie within the interval $QE_t\left(\mathcal{I}^d_{t+1}\right) = b_t x_t [L, U]$.

The Second Component of QEH

Building on Muth's insight, a QEH model represents the market's forecasts of outcomes by assuming that they lie within the intervals within which future outcomes are expected to lie, according to the qualitative expectation implied by the model.

- Representing the market's forecasts to lie within the QE intervals, but stopping short of specifying a mechanism determining the particular values that these forecasts take, is the key feature that distinguishes QEH from REH.
- While both QEH and REH rely on model consistency, QEH does so while recognizing ambiguity about the process driving outcomes.

The Model-Consistent Representation of Market Forecasts

Stock-price:

$$p_t = \gamma \left(\mathcal{F}_t \left(d_{t+1} \right) + \mathcal{F}_t \left(p_{t+1} \right) \right),$$

where $\mathcal{F}_t \left(d_{t+1} \right)$ and $\mathcal{F}_t \left(p_{t+1} \right)$ are the market's forecast of dividends and prices.

QEH represents the market's forecasts to be consistent with the qualitative predictions of the model.

To do so, we assume that the market's forecasts lie within the intervals defined by the Qualitative Expectations.

- The market's forecast of dividends:

$$\mathcal{F}_t \left(d_{t+1} \right) \in QE_t \left(\mathcal{I}^d_{t+1} \right) = b_t x_t \left[L, U \right].$$

- The market's forecast of prices:

$$\mathcal{F}_t \left(p_{t+1} \right) \in QE_t \left(\mathcal{I}^p_{t+1} \right).$$

where $\mathcal{I}^p_t$ is a no-arbitrage stochastic interval satisfying:

$$\mathcal{I}^p_t \subseteq \gamma \left(QE_t \left(\mathcal{I}^d_{t+1} \right) + QE_t \left(\mathcal{I}^p_{t+1} \right) \right).$$

The Stock Price

Iterating the QE-intervals for future prices, we show that

$$p_t = \gamma \left(\mathcal{F}_t \left(d_{t+1} \right) + \mathcal{F}_t \left(p_{t+1} \right) \right) \in \mathcal{I}_t^p,$$

where the **no-arbitrage interval** $\mathcal{I}_t^p$ satisfies:

$$\mathcal{I}_t^p \subseteq \sum_{k=1}^{\infty} \gamma^k QE_t^{(k-1)} \left(\mathcal{I}_{t+k}^d \right) = \sum_{k=1}^{\infty} \gamma^k QE_t \left(\mathcal{I}_{t+k}^d \right) = b_t x_t [L_\gamma, U_\gamma]$$

with $L_\gamma = \gamma L / (1 - \gamma L)$ and $U_\gamma = \gamma U / (1 - \gamma U)$.

We can write:

$$p_t = \theta_t x_t, \quad \text{where } \theta_t \in b_t [L_\gamma, U_\gamma].$$

Due to unforeseeable change there is no mechanism determining the value of θ_t within the interval.

We can impose restrictions on the interval for θ_t given θ_{t-1}, as we illustrate later.

Efficient Market Hypothesis Does Not Follow Under QEH

Under QEH, the stock price,

$$p_t = \theta_t x_t, \quad \text{where } \theta_t \in b_t \left[L_\gamma, U_\gamma \right].$$

The perfect-foresight price equals the present value of *actual* future dividends:

$$p_t^F = \sum_{i=1}^{\infty} \gamma^i d_{t+i} = \sum_{i=1}^{\infty} \gamma^i \left(b_{t+i} x_{t+i} + \varepsilon_{t+i} \right),$$

The QEH stock price can be written as:

$$p_t = p_t^F + \left(\theta_t - \sum_{i=1}^{\infty} \gamma^i b_{t+i} \right) x_t + \eta_t, \quad \text{where } E\left(\eta_t | x_t \right) = 0.$$

- Only if $\theta_t = \sum_{i=1}^{\infty} \gamma^i b_{t+i}$ would the market allocate resources nearly perfectly.
- $(\theta_t - \sum_{i=1}^{\infty} \gamma^i b_{t+i}) x_t$ is unforeseeable and represents Knightian uncertainty.

The failure of the Efficient Market Hypothesis is a consequence of unforeseeable change – not solely of asymmetric information, as is often supposed.

QEH Econometrics

QEH Econometrics – I

Recall the simple QEH model:

$$d_t = b_t x_t + \varepsilon_t, \qquad b_t \in \mathcal{I}_t^b \text{ (MC)}, \qquad E\left(x_t | x_{t-1}\right) = x_{t-1} > 0.$$

Challenge: For a given sample period $\{1, ., t, .., T\}$ of observations $(d_t, x_t)_{t=1}^T$ formulate an econometric model that:

1. Embeds the empirical time-series behavior of $(d_t, x_t)_{t=1}^T$, which must be verified.
2. Allows for verification of key implications of the QEH, such as:
   - $b_t > 0$ and b_t allowed to be time-varying.
   - b_t satisfies MC (in periods).

The goal of the econometric analysis is to uncover qualitative regularities:

- For example, that earnings have a positive effect on dividends and stock prices over time, though the quantitative impact changes over time in unforeseeable ways.

QEH Econometrics – II

QEH requires an econometric approach that recognizes the importance of unforeseeable structural change in the parameters of the econometric model.

- QEH implies that any econometric model will eventually cease to be relevant as the sample period is extended.
- QEH theory does not predict the timing or impact of unforeseeable change.

We discuss different regression-type models that, for some sample period, represent b_t with time-varying coefficients β_t:

$$d_t = \beta_t x_t + u_t.$$

We propose considering **random coefficient autoregressive (RCA) type-models**.

Alternatives include:

- A model with β_t piecewise constant with structural breaks.
- Rolling-window type estimation of time-varying β_t.

QEH Econometrics – III

Example of a random coefficients autoregressive (RCA) model:

$$\begin{aligned} d_t &= \beta_t x_t + u_t, \\ \beta_t &= \omega + \phi\beta_{t-1} + \alpha\frac{d_{t-1}}{x_{t-1}} \end{aligned}$$

Note that if $\phi = \alpha = 0$, then β_t is constant.

Empirically flexible and:

- Can assess the model's adequacy with misspecification tests.
- Can assess qualitative regularities:
  - $\hat{\beta}_t > 0$ at all points in time, though the size of $\hat{\beta}_t$ changes over time.
  - If $\hat{\beta}_t$ lies in the equivalent assumed stochastic interval for b_t (moderate change).

QEH Econometrics – IV

Example of a random coefficients autoregressive (RCA) model:

$$\begin{aligned} d_t &= \beta_t x_t + u_t, \\ \beta_t &= \omega + \phi \beta_{t-1} + \alpha \frac{d_{t-1}}{x_{t-1}} \end{aligned}$$

Considerations:

- Quasi-Likelihood-based estimation:

$$L_T\left(\omega, \phi, \alpha, \sigma_u^2\right) = \sum_{t=1}^{T} \left(\log \sigma_u^2 + \left(d_t - \beta_t x_t\right)^2 / \sigma_u^2 \right)$$

- Asymptotic theory non-standard (Markov chain theory).
- Bootstrap methods can be applied for statistical inference.

Simulating QEH data

We simulate dividends and earnings for a limited sample period characterized by moderate change. To do so, we must pick specific values of the parameters $\{b_1, b_2, ..., b_T\}$ within the stochastic intervals:

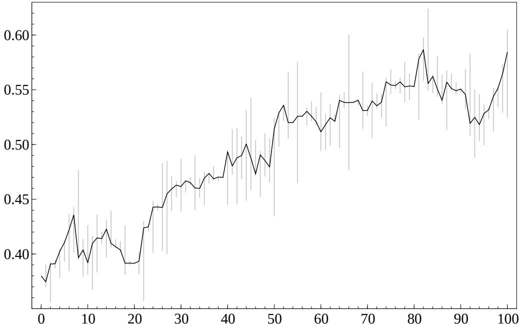
$$d_t = b_t x_t + \varepsilon_t, \qquad b_t \in \mathcal{I}_t^b = \left[b_{t-1} \left(1 - \frac{\Delta x_t}{x_t} \right)^+, b_{t-1} \left(1 + \frac{\Delta x_t}{x_t} \right) \right].$$

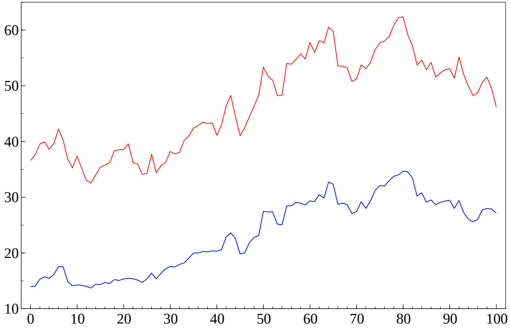
The QEH model is compatible with any sequence $\{b_1, b_2, ..., b_T\}$ satisfying this interval condition. In this sense, it is genuinely open to the unfolding of history.

1. We can manually pick one sequence $\{b_1, b_2, ..., b_T\}$.
2. Or, we can use the computer to draw the sequence $\{b_1, b_2, ..., b_T\}$ randomly from the class of stochastic models where: $b_t \sim$ Distribution over $\mathcal{I}_t^b$. We can assume uniform, normal, beta distributions, or changing distributions over time, and consider the impact of changing this distribution.

We present an illustration of simulated dividends and earnings $(d_t, x_t)_{t=1}^T$, where $\{b_1, b_2, ..., b_T\}$ is drawn uniformly over the interval $\mathcal{I}_t^b$.

Simulation Illustration of Dividends and Earnings





(A) The figure shows the simulated b_t and intervals $\mathcal{I}_t^b$ (grey vertical lines).

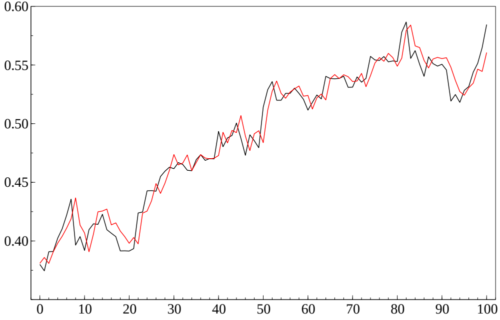
(B) The figure shows the simulated earnings x_t (red line) and dividends $d_t = b_t x_t + \epsilon_t$ (blue line).

Illustration of Econometric Modeling with Simulated Data

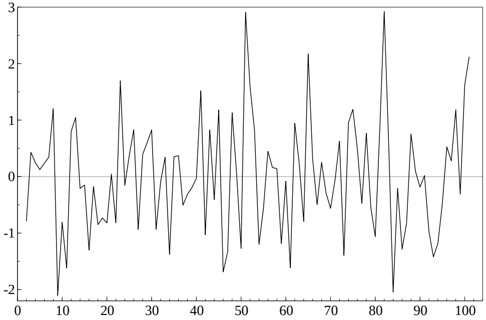
For the simulated data $(d_t, x_t)_{t=1}^{T}$, we estimate the RCA model:

$$d_t = \beta_t x_t + u_t, \qquad \text{where } \widehat{\beta}_t = 0.01 - 0.83 \cdot \widehat{\beta}_{t-1} + 0.81 \cdot \frac{d_{t-1}}{x_{t-1}}$$

$\widehat{\beta}_t > 0$ in all observations and satisfies the MC condition in 71% of the observations.



(C) The figure shows the simulated b_t (black line) and the estimates $\widehat{\beta}_t$ (red line).



(D) The figure shows the estimated residuals $\widehat{u}_t$ (standardized).

Fundamental and Psychological Factors in Driving Stock-Prices

Psychological Factors in a Behavioral-Finance Model

Behavioral-finance has emphasized the important role of **psychological factors**, but these are seen as a symptom of **gross irrationality**.

Stylized example motivated by Barberis, Shleifer, and Vishny (1998):

- Dividends process:

$$d_t = bx_t + \varepsilon_t.$$

- Let s_t be a market sentiment index with two states: pessimism when $s_t = 0$ and optimism when $s_t = 1$.
- Behavioral-finance models represent the market's forecasts to depend on sentiment in a way that is consistent with the disciplinary consensus. That is, they specify a complete stochastic process driving outcomes:

$$\begin{aligned} \mathcal{F}_t\left(d_{t+1}|x_t, s_t = 0\right) &= B_0 x_t, \quad \text{where } B_0 < b, \\ \mathcal{F}_t\left(d_{t+1}|x_t, s_t = 1\right) &= B_1 x_t, \quad \text{where } B_1 > b. \end{aligned}$$

- The market's forecasts are inconsistent with the quantitative prediction of the model, which leads to systematic forecast errors. Market participants are viewed as grossly irrational, while $E(d_{t+1}|x_t) = bx_t$ is the only rational forecast.

Diversity without Irrationality

Opening a model to unforeseeable change allows a QEH model to incorporate psychological influences without assuming gross irrationality.

- A QEH model makes *qualitative predictions*: future outcomes are expected to lie within stochastic intervals.
- Consequently, myriad possible quantitative forecasts are consistent with the process the economist assumes drives outcomes.
- Thus, diversity does not imply gross irrationality.

Psychological Factors in the QEH Model

Rational market participants facing ambiguity select particular quantitative forecasts by relying on a combination of formal (econometric) models, market sentiment, and other non-fundamental factors.

A QEH model formalizes the *qualitative effect* of such factors on participants' model-consistent forecasts by imposing additional restrictions on how they revise the weighting of fundamentals over time.

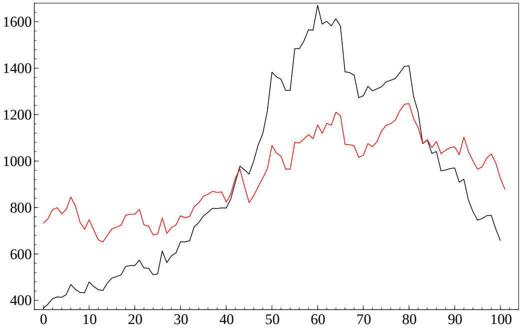
- For example, in addition to $\mathcal{F}_t\left(d_{t+1}\right) \in QE_t\left(\mathcal{I}_{t+1}^d\right) = b_t x_t\left[L, U\right]$, we assume that the interval for the market's forecast depends on sentiment:

$$
\begin{aligned}
\mathcal{F}_t\left(d_{t+1} | s_t = 0\right) &= \tilde{b}_t x_t, \quad \text{where } \tilde{b}_t \in b_t\left[L, 1\right], \\
\mathcal{F}_t\left(d_{t+1} | s_t = 1\right) &= \tilde{b}_t x_t, \quad \text{where } \tilde{b}_t \in b_t\left[1, U\right],
\end{aligned}
$$

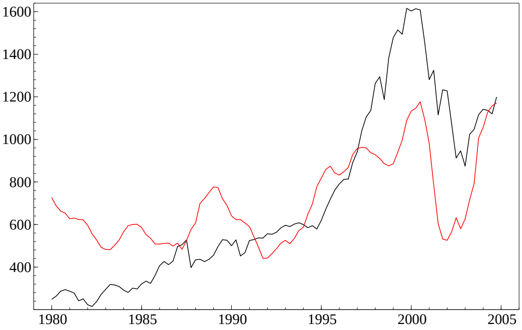
  where we interpret $\tilde{b}_t$ as the market's forecast of b_{t+1}.

- When the market is optimistic, it forecasts b_{t+1} to be higher than b_t. When the market is pessimistic, it forecasts b_{t+1} to be lower than b_t.

Prices and Earnings: Simulated and S&P500



(E) The figure shows the simulated price, p_t (black line), and earnings, x_t, multiplied by 20 (red line).



(F) The figure shows the S&P500 stock index (black line) and company earnings multiplied by 20 (red line).

Concluding Remarks

We have presented the Qualitative Expectations Hypothesis (QEH) in the context of a simple stock-price model.

Much work remains to be done to determine if QEH can shed light on the long-standing puzzle of what drives stock-price movements.

However, we believe that opening economic models to unforeseeable change is crucial for understanding how well asset markets allocate society's savings and what role the state might play in regulating them.

Concluding Remarks – II

Despite its simplicity, the QEH model presented today captures key features of models typically used in other contexts.

- For example, forward-looking expectations in the New Keynesian approach that underpins the DSGE models used by central banks.

One area of future research is to assess whether QEH's approach to formalizing the inherent ambiguity that policymakers and market participants face could help us resolve some of these models' empirical difficulties, and thereby enhance macroeconomic models' usefulness for policy analysis.

Concluding Remarks – III

QEH offers a way to formalize the limits of what we can know about the future.

Consensus models assume that the future is exactly the same as the past.

- Consequently, as more data becomes available, our understanding of quantitative regularities should become more precise.

Unforeseeable structural change implies that the future is different from the past.

- As more data becomes available, QEH predicts that any model undergoes structural change.
- Knight's "problem of knowledge": quantitative regularities out of reach for economic analysis.

Concluding Remarks – IV

Almost a century ago, Knight elegantly summarized the problem of knowledge:

> *We live in a world full of contradiction and paradox, a fact of which perhaps the most fundamental illustration is this: that the existence of a problem of knowledge depends on the future being different than the past, while the possibility of the solution of the problem depends on the future being like the past.*

Potential solution to the knowledge problem:

- Qualitative regularities characterizing past outcomes also characterize future outcomes.
- Testing whether this is the case suggests the need for a theoretical and econometric approach like the one we presented this afternoon.